Quantitative Selection of Variation Reduction Plans

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Quality has been a rallying call in design and manufacturing for the last two decades. One way to improve quality is through variation reduction (VR). VR teams use tools such as Design of Experiments (DoE) and robust design to improve product performance and quality by reducing variation introduced by manufacturing processes. Because VR teams are typically resource constrained, they must carefully select where to focus their efforts. Planning for VR is complex because reduction efforts are executed on individual features and processes but benefits are accrued when the overall product quality improves. The problem is further complicated by the existence of multiple performance criteria and hundreds of processes and dimensions that effect each performance requirement. Consequently, VR teams typically use qualitative assessments to prioritize and schedule their efforts. This paper provides a mathematical model capable of optimally allocating VR resources for a complex product. The VR model has three parts: a model of variation propagation, a model of variation costs, and a model of variation reduction costs. These models are used to directly calculate the optimal resource allocation plan and schedule for a product with multiple product quality requirements. An example from the aerospace industry is used to demonstrate the theory. [S1050-0472(00)00602-4]

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1 Introduction

Quality has been a rallying call in design and manufacturing for the last two decades. This all encompassing term can be applied to many aspects of a product: design quality (e.g., shape of a car or features on a copier), distribution and service quality (e.g., the experience the customer has purchasing, bringing home, and setting up a product), performance quality (e.g., how the product works in a various environments), and manufacturing quality (e.g., how much manufacturing variation affects performance of a product). Companies are still struggling with all, especially the later. The impact of variation on profitability has been the topic of many articles in the academic and popular presses [1–3]. However, several studies indicate that most quality initiatives fail to significantly improve quality [4].

Proponents of quality initiatives, such as Deming [5] and Taguchi and Clausing [6], have asserted that quality-efforts will always have a positive return: i.e., quality is always free. In addition, others [7–9] have proposed methods to set optimal quality levels by balancing quality improvements against their cost. The latter methods model the quality for a single quality/product characteristic and a single control feature. However, a design team is rarely faced with such a simple problem. In most cases, they are faced with multiple control options and multiple quality targets.

In a complex product, a variation reduction team (described in detail in Section 1.3) must choose between improving multiple product quality measures. They must also decide what processes to improve because every product quality measure is influenced by multiple part and process characteristics. While most projects will return a profit, (i.e., quality is free), projects should be selected to make the most *effective* use of a limited resource set.

Currently, the choice, order, and scheduling of projects are based on qualitative measures.[1] While models are available to evaluate and optimize design alternatives [10,11] and process alternatives [12], a few models are capable of evaluating the costs and benefits of variation reduction plans and identifying the optimal plan. For example, Atwater and Chakravorty [4] propose a method to identify the optimal quality improvement plan for a production system. However, their work focuses on reducing scrap to improve throughput rather than improving the overall product quality and do not include the cost of improvements in their calculations.

Effective prioritization requires modeling the source of variation, the impact of variation on product performance, the cost of variation, and the cost of variation reduction. The net present value (NPV) should be used to calculate a more accurate measure of benefit. In addition, the effect of project scheduling should be modeled.

1.1 Problem Statement. Figure 1 illustrates the problem space. At the center, the product model predicts product performance from the part and process variation. At the top of the figure, degradation in product performance results in an increased cost of variation. The bottom of the figure shows the cost of variation reduction. When processes are improved, variation reduction costs should be balanced against the reduced cost of variation. Prioritizing variation reduction is complex because of multiple product quality requirements, significant numbers of processes, and complex cross-interactions between the two.

1.2 Key Characteristics. In a complex product, it is not economically or logistically feasible to control and/or monitor thousands of tolerances and processes. To identify what tolerances and processes to control, many organizations are using a method called Key Characteristics (KCs) (also termed Critical Parameters and Special Characteristics). KC methods are used by design to identify and communicate to manufacturing where excess variation will most significantly affect product quality [13].

First, design teams identify where variation may impact performance, safety, and fit requirements. Performance requirements are determined by the market and customers, safety requirements are set by internal and external (i.e., FAA) safety groups, and fit requirements are issues that can make manufacture and assembly difficult (e.g., loading a part into a fixture). The product quality requirements are called *product-KCs*.

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[1] Generalizations are based on observations by the author of a variety of companies and their practices. Due to proprietary issues, specific names and practices are not given.

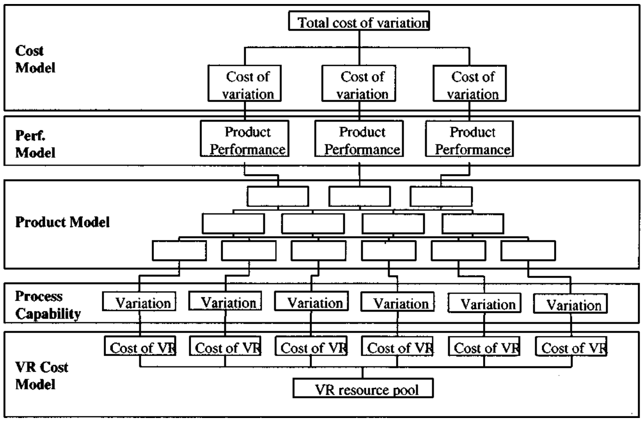

Fig. 1 Variation and Variation Reduction Cost Model

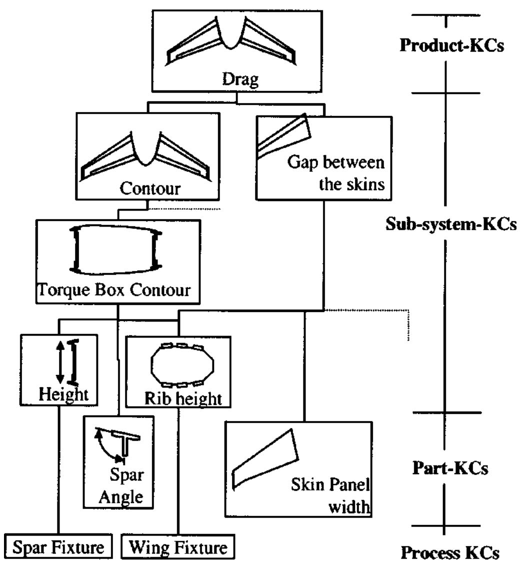

Fig. 2 KC Flowdown

Second, the design team flows these requirements through the product decomposition. Figure 2 shows a simplified KC flowdown for the drag on an aircraft horizontal stabilizer. The product-KC (drag) at the top of the tree has several contributing subsystem-KCs (contour and gap). The subsystem-KCs are, in turn, a function of the part-KCs (spar angle) and process-KCs (fixtures). In some cases, it may be possible to flow the KCs down to the process level; however, in some cases parts are delivered by outside suppliers. In this case, part-KCs become product-KCs for the supplier. It is assumed here that control is imposed at the process level.

In summary, the KC flowdown provides a system view of potential variation risk factors and captures the design team's collective knowledge about variation and its contributors. Traceability between process- and product-KCs through part- and subsystem-KCs is necessary to identify what processes should be controlled, identify where process change may positively impact product quality, and identify root causes of quality problems.

1.3 Variation Reduction. Variation Reduction (VR) is a term applied to a broad set of tasks performed by process improvement teams. VR works to reduce variation in processes causing significant excess cost. VR teams are usually responsible for continual process improvement once a product is in production. VR efforts are important because cost reductions in production translate into pure profit or an ability to drop product price.

VR teams typically have a fixed number of people and a fixed budget which limits the number and scope of possible projects. In general, they firefight problems causing the largest pain. VR target areas are often identified by other function groups. For example, manufacturing will highlight areas with unacceptable levels of rework, scrap, repair, or labor.

Once an area is targeted, VR teams perform root cause analyses to identify contributors to the problem. Using tools such as Failure Modes and Effects Analysis (FMEA), Design of Experiments (DOE) and Variation Simulation Analysis (VSA),[2] VR teams prioritize the contributing factors. The contributors can be dimensions, processes, or physical attributes (i.e., electrical characteristics). A few of the contributors are subjected to improvement efforts: changing process parameters, upgrading machines, machine maintenance, and/or implementation of standard processes and employee training.

Most VR teams are very effective at removing variation from targeted areas. However, they tend to use qualitative methods to prioritize their efforts. To optimize the allocation of limited resources, a quantitative method is needed to enable teams to rapidly prioritize where resources are best allocated as well as to optimally schedule projects.

1.4 Paper Structure. This paper presents a method to optimize VR plans. Figure 1 shows the framework for the mathematical model. First, the center section of the figure is derived. This model enables the product-KC variations to be calculated from process-KC variations (Section 3). Next, the cost of variation is modeled (Section 4). Third, the cost of variation reduction is modeled (Section 5). Fourth, the optimization model is presented (Section 6). Fifth, the concept of NPV is presented (Section 7). Finally, the implications of scheduling on VR effectiveness are discussed (Section 8). The tools are illustrated using the case study introduced in Section 2.

2 Case Example

This section describes the case, the assembly of an aircraft wing center box, used to illustrate VR optimization.[3] This case was first introduced in [14]. The case has been expanded to include the processes used to create the feature-KCs.

An aircraft wing has three main sections (shown in Fig. 3), the center box, leading edge, and trailing edge. The center box, the focus of this case, provides structural stiffness, surface area, and attachment to the fuselage. The wing has many variation sensitive requirements including the steps and gaps between skin panels, wing orientation, and upper and lower surface contour.

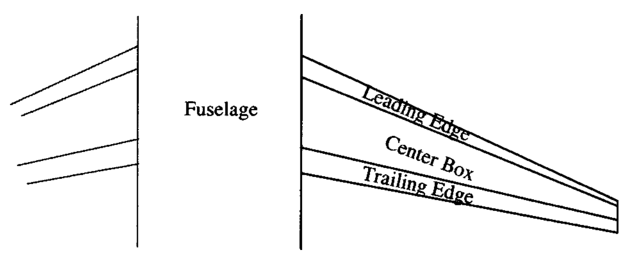

Fig. 3 Wing configuration

[2]Variation Simulation Analysis is a computational tool that uses Monte Carlo simulation and a description of the geometry to model the effect of part variation on the system.

[3]The assembly description and process steps are consistent with current aircraft assembly processes, but details, capability, and geometry are modified and approximated to mask any proprietary data. Because the numbers are changed, the conclusions of capability and high risk KCs do not reflect the actual capability in aircraft manufacture.

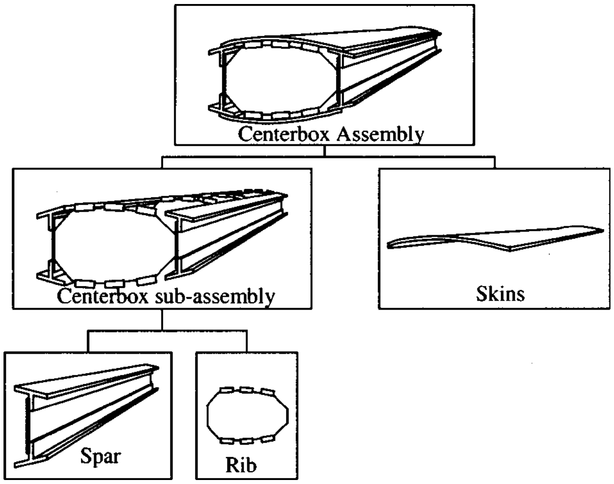

Fig. 4 Center box assembly process

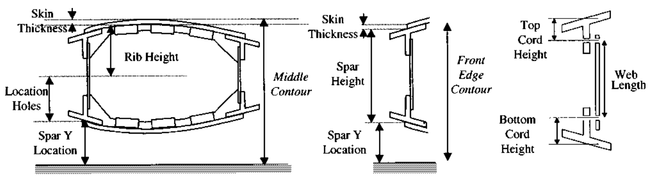

Fig. 5 Tolerance stack-ups for leading edge and middle contour

This case focuses on the contour of center box surface. The center box has three main sub-assemblies: spars, skins, and ribs. The assembly tree is shown in Fig. 4. The three sub-assemblies are assembled separately in their own fixtures and then the center box is built in the wing fixture. The spars are placed in the fixture first. The spars are assembled from two cords (T-shaped extrusions) and a web (a reinforced aluminum sheet) and are assembled using a determinate assembly process (parts are located relative to each other using precision location holes [15]). The ribs are assembled in their own fixture and are located relative to the spars using locating holes in the spar web. Finally, skins are attached to the top and bottom surfaces defined by the ribs and spars.

This case study focuses on two wing KCs: the leading edge and middle contours as measured from aircraft coordinates. The product-KCs are flowed down to the process-KCs by analyzing the tolerance stack-ups. The tolerance paths are shown in Fig. 5.

The contours are a function of several subsystem-KCs including the error in the location fixture used to locate the spars (*spar fixture: y loc*), the rib height, rib location holes, skin thickness, and spar heights. The skin thickness is a function of the process used to form the aluminum sheet (*al sheet: thickness*). The rib location holes are a function of the process used to drill holes in the aluminum sheet (*al sheet: loc. holes*). Because the ribs are built in a fixture, their height is a function of the rib fixture capabilities (*rib fixture: height*). The spar height is a function of the process used to drill holes in the cords (*al extrusion: loc holes*) and the process used to drill holes in aluminum sheet (*al sheet: loc. holes*).

3 Variation Propagation Model

The variation propagation model predicts product-KCs' standard deviations (i.e., the contour) from the process-KCs' standard deviations. This model is based on a linearized model of variation described in Thornton [16]. While models such as Variation Systems Analysis (VSA) can be used to predict capability more accurately, the modeling and analysis time for them are typically significantly longer.

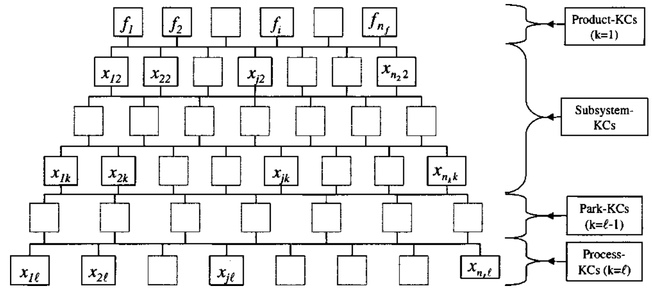

Fig. 6 Generalized KC flowdown

3.1 Math Model. A mathematical model for variation propagation was proposed by the author in [16]. Figure 6 is a generalized version of the KC flowdown in Fig. 2. The top layer of the flowdown has n_f product-KCs. Each has a nominal value, f_i, a tolerance, $[LL_i, UL_i,]$, a standard deviation, σ_i, a mean, μ_i, and a failure cost, C_i. Product-KCs are created by l layers of subsystem-, part-, and process-KCs. Each layer, k, has n_k KCs.

Using Taylor expansions, the expression relating the deviation in a product-KC, Δf_i,[4] to the next layer of KCs, is represented as a linear equation.

$$\Delta f_i = \frac{\partial f_i}{\partial x_{12}} \Delta x_{12} + \frac{\partial f_1}{\partial x_{22}} \Delta x_{22} + \cdots + \frac{\partial f_1}{\partial x_{n_2 2}} \Delta x_{n_2 2} \tag{1}$$

$\partial f_i / \partial x_{j2}$ captures the sensitivity of product-KC, f_i, to a change in subsystem-KC, x_{j2}. The sensitivities can be derived using DOE [17], tolerance chains [12], or modeling [18]. The array subsystem-KC deviations, $\mathbf{\Delta x_2}$, can be related to the product-KCs array, $\mathbf{\Delta f}$, through a matrix of partial derivatives, δ_1. A matrix approach to variation propagation and modeling has been used extensively by authors such as Homann and Thornton [19], Slocum [20], Suh [21], Frey et al. [22] and Gao et al. [23].

$$\mathbf{\Delta f} = \boldsymbol{\delta}_1 \mathbf{\Delta x_2} \quad \text{where} \quad \boldsymbol{\delta}_1 = \begin{bmatrix} \frac{\partial f_1}{\partial x_{12}} & \frac{\partial f_1}{\partial x_{22}} & \cdots & \frac{\partial f_1}{\partial x_{n_2 2}} \\ \frac{\partial f_2}{\partial x_{12}} & \frac{\partial f_2}{\partial x_{22}} & \cdots & \frac{\partial f_2}{\partial x_{n_2 2}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_{n_1}}{\partial x_{12}} & \frac{\partial f_{n_1}}{\partial x_{22}} & \cdots & \frac{\partial f_{n_1}}{\partial x_{n_2 2}} \end{bmatrix} \tag{2}$$

For each subsequent layer, δ_1 can be generalized to the matrix, $\boldsymbol{\delta}_\mathbf{k}$, which relates $\mathbf{\Delta x_{k+1}}$ to $\mathbf{\Delta x_k}$.

$$\mathbf{\Delta x_K} = \boldsymbol{\delta}_\mathbf{K} \mathbf{\Delta x_{(K+1)}},$$

where

[4] $\Delta f = \Delta x_1$, but we are using f to be consistent with other literature (Srinivasan, 1998).

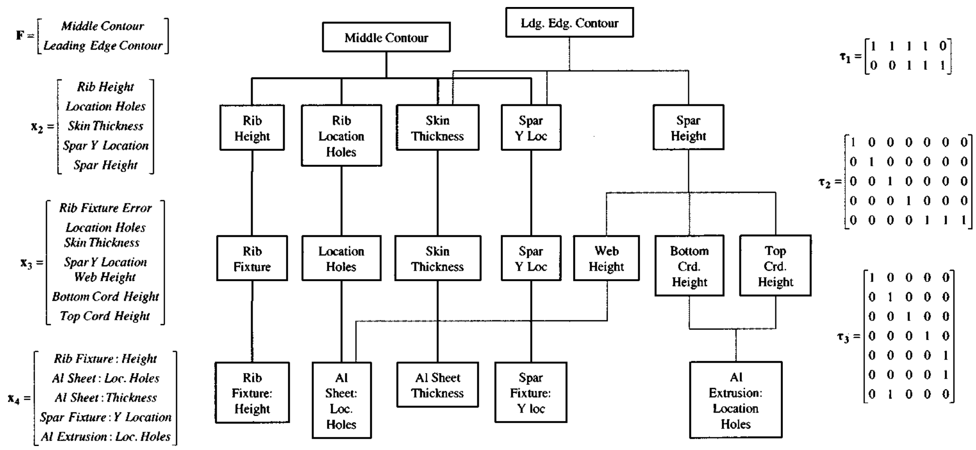

Fig. 7 KC Flowdown for wing contour

$$\boldsymbol{\delta}_{\mathbf{K}}=\begin{bmatrix} \frac{\partial x_{1k}}{\partial x_{1(k+1)}} & \frac{\partial x_{1k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{1k}}{\partial x_{n_{k-1}(k+1)}} \\ \frac{\partial x_{2k}}{\partial x_{1(k+1)}} & \frac{\partial x_{2k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{2k}}{\partial x_{n_{k-1}(k+1)}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial x_{n_k k}}{\partial x_{1(k+1)}} & \frac{\partial x_{n_k k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{n_k k}}{\partial x_{n_{k-1}(k+1)}} \end{bmatrix} \quad (3)$$

The correlation between variation in process-KCs (represented by array $\Delta\mathbf{x}_l$) and product-KCs (represented by array $\Delta\mathbf{f}$) can be expressed by:

$$\Delta\mathbf{f}=\mathbf{D}\Delta x_l \quad \text{where} \quad \mathbf{D}=\boldsymbol{\delta}_1\boldsymbol{\delta}_2\boldsymbol{\delta}_3\ldots\boldsymbol{\delta}_{(l-1)} \quad (4)$$

Equations (1)–(4) model errors for a single product. In addition, the statistical behavior of errors can be modeled. The partial differentials are used to calculate the standard deviation, σ_{jk}, for any KC, x_{jk}.

$$\sigma_{jk}^2=\left(\frac{\partial x_{jk}}{\partial x_{1(k+1)}}\right)^2\sigma_{1(k+1)}^2+\left(\frac{\partial x_{jk}}{\partial x_{2(k+1)}}\right)^2\sigma_{2(k+1)}^2+\cdots+\left(\frac{\partial x_{jk}}{\partial x_{n_{(k-1)}(k+1)}}\right)^2\sigma_{n_{(k-1)}(k+1)}^2 \quad (5)$$

Equation (5) can also be represented in a matrix form but a second matrix, $\tau_{\mathbf{k}}$, is needed.

$$\boldsymbol{\tau}_{\mathbf{K}}=\begin{bmatrix} \left(\frac{\partial x_{1k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{1k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{1k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \\ \left(\frac{\partial x_{2k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{2k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{2k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial x_{n_k k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{n_k k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{n_k k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \end{bmatrix} \quad (6)$$

When Eqs. (5) and (6) are combined the product variation can be predicted.

$$\boldsymbol{\sigma}_{\mathbf{f}}^2=\mathbf{T}\boldsymbol{\sigma}_l^2 \quad \text{where} \quad \mathbf{T}=\boldsymbol{\tau}_1\boldsymbol{\tau}_2\boldsymbol{\tau}_3\ldots\boldsymbol{\tau}_{(l-1)} \quad (7)$$

$\sigma_{\mathbf{f}}$ is the array of product-KC standard deviations and $\boldsymbol{\sigma}_l$ is the array of process- and part-KCs standard deviations. In most cases $D_{ij}^2=T_{ij}$ except when a part- or process-KC contributes multiple times to the same product-KC. This may happen, for example, if a

Table 1 Failure index as a function of *Cpk*

Range	*Failure index*
Cpk<1	**Red**
1<Cpk<1.33	**Yellow**
1.33<Cpk	**Green**

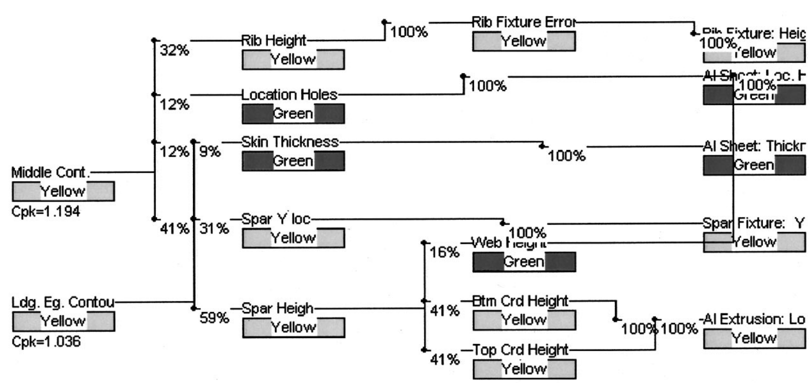

Fig. 8 Results from KCTool

single worn tool is used to make multiple parts for an assembly and, consequently, all parts in the assembly are undersized.

3.2 Example. Figure 7 shows the KC flowdown for the contour example (Section 2) with the τ matrices. The top of the flowdown contains two product-KCs: middle contour and leading edge contour. The second layer contains the subsystem-KCs. The third layer contains the individual part-KCs. At the lowest level, the process-KCs are listed.[5] In two instances, one process-KC influences more than one part-KC. For example, the process capability for drilling location holes in aluminum extrusions (*al extrusion: loc holes*) contributes to both the bottom and top cord heights. Reducing variation in hole locations improves two part-KCs. This effect was pointed out recently by one of our industrial partners ''We build thousands of different parts but use only 20 processes . . . if we improve each process, we influence thousands of parts.''

The final matrices are

$$\mathbf{D}=\begin{bmatrix}1&1&1&1&0\\0&1&1&1&2\end{bmatrix},\quad \mathbf{T}=\begin{bmatrix}1&1&1&1&0\\0&1&1&1&2\end{bmatrix} \tag{8}$$

In this case, all sensitivities are derived from one-dimensional tolerance stackups. In cases where the stackups are two-dimensional, the elements in the **D** and **T** matricis will be more complex to derive. However, variation analysis software such as VSA can be used to calculate sensitivities directly.

In order to visual display of KC flowdowns, a proof-of-concept software tool, *KCTool* has been developed [24,14]. The data model is based on a hierarchical description of variation, also termed a *Key Characteristic (KC) flowdown*. KCTool was developed to provide an experimental environment in which analysis tools could be built and tested [25,14]. To visually display the risk, color and numerical data are layered on the KC flowdown. The visual display is generated in three steps. First, the product-KCs are assigned a failure index based on their allowable limits and their predicted σ and μ values. Second, the index is mapped to three colors: green, yellow, and red based on the values in Table 1. Third, failure indices are assigned to each child based on three factors: the product-KC failure index, number of children, and contribution factor of each child KC. The percentage contributions are determined based on the relative contribution of the bias and standard deviation to the parent-KC's overall bias and standard deviation. Details of the color and percent assignment algorithm can be found in Thornton [14].

The results in Fig. 8 show that the leading edge contour is at higher risk than the middle contour. In addition, location holes in the aluminum extrusions, the spar fixtures, and the rib fixtures contribute more than sheet location holes and sheet thicknesses.

4 Model of Variation Costs

Costs associated with variation can be roughly classified into two categories: external costs and internal costs. External costs are generated after the product leaves the manufacturing site and include product returns and product repair. Internal costs are generated before the product leaves the plant and include rework and scrap costs. In addition, other less visible internal costs result from production disruptions, increased inspection requirements, under utilization of manufacturing capability, and production ramp-up delays. Because it is not possible to model all cost contributors, most companies use the Taguchi loss function to model the cost of variation. Although the Taguchi function tends to overestimate variation costs, it does provide a good measure of relative costs.

4.1 Math Model. The cost of variation, L_i, incurred when product-KC i deviates from nominal by an amount Δf_i is

[5]The process-KCs are named using the format *material/process: feature type*. The first indicates the material or process and the second the feature type. This kind of notation is typical in process capability databases.

Table 2 Process capability

	σ_i
Rib Fixture: Height	0.008
Al Sheet: Loc. Holes	0.005
Al Sheet: Thickness	0.005
Spar Fixture: Y location	0.009
Al Extrusion: Loc. Hole	0.008

$$L_i=k_i(\Delta f_i)^2 \quad \text{where} \quad k_i=C_i\Big/\left(\frac{UL_i-LL_i}{2}\right)^2 \tag{9}$$

where C_i is the cost of not achieving the tolerance $[LL_i, UL_i]$.

For a population of products, the expected loss for product-KC i is a function of its standard deviation, σ_i, and cost constant, k_i. In turn, the product-KC's standard deviation is a function of T_{ij} (defined by Eq. (7)) and the process-KC standard deviations, σ_j.

$$L_i=k_i(\sigma_i^2)=k_i\left(\sum_{j=1}^{n_l} T_{ij}\sigma_j^2\right) \tag{10}$$

In most cases, a product has multiple product-KCs ($n_f>1$ and a single process-KC may contribute to more than one product-KC). The total cost of variation, L_T, is the sum of the individual loss functions, L_i.

$$L_T=\sum_{i=1}^{n_f} L_i=\sum_{i=1}^{n_f} k_i\sigma_i^2=\sum_{i=1}^{n_f} k_i\left(\sum_{j=1}^{n_l} T_{ij}\sigma_j^2\right) \tag{11}$$

Calculating the total loss is important because using L_T enables the optimization to identify where a process variation contributes to more than one product-KC.

4.2 Example. Table 2 gives the process capability for the process-KCs. The machined features (e.g., Al Sheet: Loc. holes) have smaller standard deviations than the fixture related KCs (e.g., Rib Fixture: Height).

Table 3 shows target specifications.[6] C_F is \$500, the production rate is 100 per year, and the production life is 10 years.

5 Model of VR Costs

In order to optimize a variation reduction plan, the cost of variation reduction needs to be modeled.

5.1. Math Model. This section describes how a VR plan and its costs are modeled. These models include the cost of variation, L_T. VR plan cost, I_T, variation cost after the VR plan is executed, L_T^*, and return on investment, ROI. The cost, I_j, to reduce process variation from σ_j to σ_j^* for process-KC j can be modeled as [15]:

$$I_j=\kappa_j\left(\frac{1}{\sigma_j^*}-\frac{1}{\sigma_j}\right) \quad \text{where} \quad \kappa_j=\frac{C_{dj}}{\left(\frac{1}{\sigma_{dj}}-\frac{1}{\sigma_j}\right)} \tag{12}$$

κ_j is a constant calculated by estimating the cost, I_{dj}, to reduce variation by a fixed amount, σ_{dj}. The anchor points are usually generated by estimating the cost to halve the variation. Like the Taguchi loss function, this curve provides a good measure of the trend rather than an accurate cost prediction.

In most products, multiple process-KCs contribute to a single product-KC. In addition, if a product has n_l features, only a subset of them n_l^* will be subjected to variation reduction. In addition, a fixed cost, C_F, must be spent for every feature subjected to variation reduction (the total fixed cost for VR is $n_l^* C_F$). C_F includes

[6]All tolerances are from nominal and are measured in inches.

Table 3 Product requirements

	UL_i	LL_i	C_i	k_i	L_i	Cp_i
Middle Cont.	-0.05	0.05	$200	$80,000	$15.6	1.19
Ldg. Eg. Contour	-0.05	0.05	$300	$120,000	$31.08	1.04

Table 4 Process capability and uncertainty for wing case

	σ	σ_d	*Id*
Rib Fixture: Height	0.008	0.004	$3,000
Al Sheet: Loc. Holes	0.005	0.0025	$1,300
Al Sheet: Thickness	0.005	0.0025	$7,000
Spar Fixture: Y location	0.009	0.0045	$3,000
Al Extrusion: Loc. Hole	0.008	0.004	$5,000

resources required to baseline process capability and purchase measurement equipment.[7] Assuming there are n_l process-KCs, the total product VR cost, I_T, is calculated by:

$$I_T = n_l^* C_F + \sum_{j=1}^{n_l} \kappa_j \left(\frac{1}{\sigma_j^*} - \frac{1}{\sigma_j} \right) \tag{13}$$

The total variation cost, L_T^* after improvement is calculated by:

$$L_T^* = \sum_{i=0}^{n_f} k_i (\sigma_i^*)^2 = \sum_{i=1}^{n_f} k_i \left[\sum_{j=1}^{n_l} T_{ij} (\sigma_j^*)^2 \right] \tag{14}$$

The benefit, ΔL_T, is calculated by subtracting the variation cost after improvement, L_T^* from the original variation cost, L_T.

$$\Delta L_T = L_T - L_T^* \tag{15}$$

Two methods can be used to calculate the return for the VR effort. The first takes the difference between Eqs. (13) and (15). However, calculating net profit is problematic because it is difficult to ensure that the dollars invested, I_T, are the same as the dollar benefit, ΔL_T. Because of this, it is more appropriate to use the ratio of the two measures (i.e., the *ROI*) where n is the number of products remaining in the production cycle

$$ROI = \frac{n(L_T - L_T^*)}{I_T} \tag{16}$$

In this case, an *ROI* of greater than one does not ensure a profitable project. The measure of *ROI* in Eq. (16) provides a *relative* measure of effectiveness and should be used to rank order efforts rather than making a go/no-go decision on a single project. In addition, the calculations require the user to specify the costs of variation as well as the cost of performing reduction. Because the *ROI* measures the ratio of the cost of failure to the cost of VR, it is not necessary to ensure that the values are in the same units. Because these methods are being used to rank the project, it is not necessary to model the costs and benefits precisely. However, It is necessary to ensure that the *relative* costs are correctly specified. The method presented in this paper was found to be insensitive to small changes in the costs as long as the relative ranking of costs remained the same.

5.2 Example. The cost of variation reduction used in subsequent section is shown in Table 4.

6 Optimization Plan

Assuming a VR team has a limited set of resources, I_T, the expenditures, I_j, are set to maximize total *ROI* (Eq. (16)). However, in some cases, the optimal value of I_j will be zero (i.e., some features are not improved). In Thornton [26], this optimization was performed using simulated annealing; however, that method was computationally intensive. This section proposes a method to calculate the optimal resource allocation directly with out expensive simulation.

The optimal allocation of resources for a total VR budget of I_T is calculated using a two step process. First, the optimal *ROI* for n_l cases are calculated. In the first case, the optimal *ROI* (ROI_1) is calculated assuming only one process-KC is improved. In the second case, the optimal *ROI* (ROI_2) is calculated assuming only two process-KCs are improved. In the *n*th case, the optimal *ROI* (ROI_n) is calculated assuming n process-KCs are improved.

Second, the optimal resource allocation is found by selecting the case which has the highest *ROI*:

$$ROI_{\max} = \max\lfloor ROI_1, ROI_2, \ldots, ROI_{n_l} \rfloor \tag{17}$$

The next few sections describe how the optimal value for each case is found.

$\mathbf{n_l^* = 1}$. The first case is trivial: all resources are allocated to the first KC. The value of ROI_1 is found by

$$ROI_1 = \max\left[\frac{n\Delta L_T}{I_1 + C_F}, \frac{n\Delta L_T}{I_2 + C_F}, \cdots, \frac{n\Delta L_T}{I_{n_l} + C_F} \right] \quad \text{subject to} \quad \begin{matrix} I_i + C = I_T \\ I_i > 0 \end{matrix} \tag{18}$$

n_l process-KCs are improvement candidates. Each sub-case is tested and the maximum *ROI* is used for ROI_1. Figure 9 shows a graph of Eq. (18) for the five process-KCs. Improving *Spar Fixture: Y location* process is the most effective for all values of I_j.

$\mathbf{n_l^* = 2}$. Calculating *ROI* for two KCs can also be done explicitly. Given a set amount, I_T, I_1 and I_2 are set by maximizing

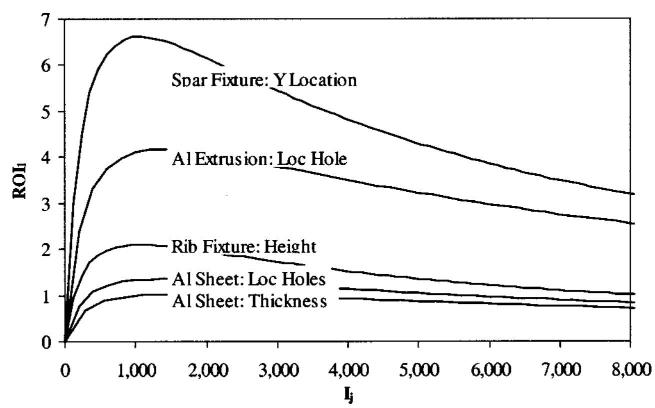

Fig. 9 *ROI* for $n_l^* = 1$

[7] In this case, C_F is assumed to be the same for all process-KCs. However, the analysis below can be done with different values C_F for each KC.

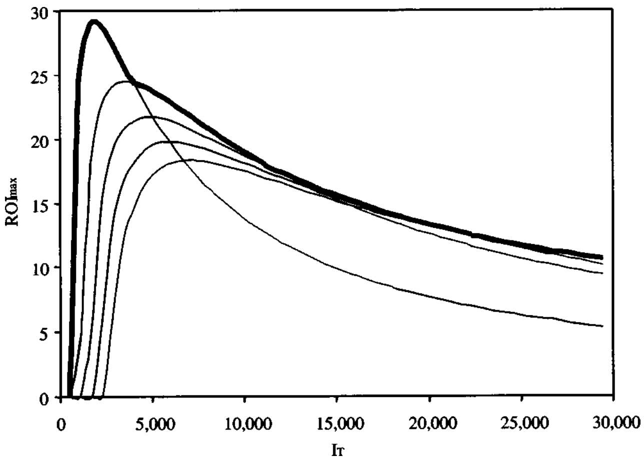

Fig. 10 ***ROI*₁, *ROI*₂, . . . *ROI*ₙ, and *ROI*ₘₐₓ**

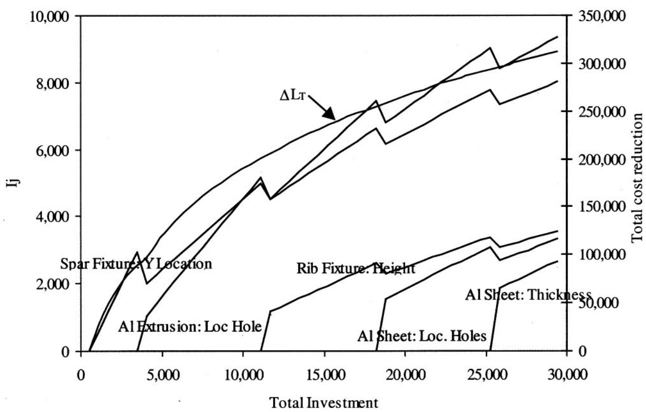

Fig. 11 Optimal distribution and marginal benefit

$$ROI_2=\frac{n\Delta L_T}{I_1+I_2+2C_F} \text{ subject to } \begin{array}{l} I_T=I_1+I_2+2C_F \\ I_1>0 \\ I_2>0 \end{array} \tag{19}$$

for $n_l(n_l-1)/2$ cases of pairs of process-KCs.

The optimal distribution for each case is found by taking the partial derivative of ROI_2 with respect to I_1 and setting the value equal to zero.

$$\frac{\partial ROI_2}{\partial I_1}=n\sigma_1^2\sum_{i=0}^{n_f} k_iT_{i1}\left(\frac{2\sigma_1\kappa_1^2}{(I_1\sigma_1+\kappa_1)^3}\right) - n\sigma_1^2\sum_{i=0}^{n_f} k_iT_{i1}\left(\frac{2\sigma_2\kappa_2^2}{[(T-2C-I_1)\sigma_2+\kappa_2]^3}\right) \tag{20}$$

The optimal value of I_1 is:

$$I_j=\frac{\beta\lfloor(T-2C)\sigma_2+\kappa_2\rfloor-\kappa_1}{(\sigma_1+\beta\sigma_2)} = \text{where } \beta=\frac{\sigma_1}{\sigma_2}\sqrt[3]{\frac{\kappa_1^2\sum_{i=0}^{n_f} k_iT_{i1}}{\kappa_2^2\sum_{i=0}^{n_f} k_iT_{i2}}} \tag{21}$$

$\mathbf{n_l^*=n}$. The third and subsequent cases cannot be calculated directly because the optimization occurs over multiple characteristics. Because *ROI* is continuously concave for each I_j, the maximum *ROI* can be found using simple optimization routines. In this case, a modified simplex search was used to find the optimal value for

$$ROI_n=\frac{n\Delta L_T}{I_1+I_2+\ \ldots\ +I_n+nC_F} \tag{22}$$

For each case, $n_l!/(n_l-n)!n!$ must be tested.

***ROI*$_{\max}$**. Figure 10 shows charts for all five cases with *ROI*$_{\max}$ highlighted in gray. In this example, there is very little difference between improving three, four, or five process-KCs.

Figure 11 shows the optimal distribution of resources to each KC for each value of I_T. For example, assuming a total investment of 15K, approximately 2K would be invested in reducing the variation in the rib fixture height, 6K in the Spar Fixture Y location and the remainder in the location hole in the extrusion. The investment curves are jagged because each time an additional feature is added to the VR plan, the available resources drops (because of the additional fixed cost to start the next project).

In addition, the total cost reduction, ΔL_T, is plotted. It is clear from the graph that there is a decreasing marginal return for increased investment. To gain the first 100K benefit, it costs 4K, the next 100K costs an additional 7K and the third 100K costs an additional 15K.

Order of inclusion. Evaluating all of the cases and sub-cases can be time consuming to execute when the value of n_l is large. To avoid this, KCs can be ranked according max(ROI_I). The process-KCs with a higher maximum *ROI* are more cost effective and should be included first. The *ROI* for a single feature is:

$$ROI_j=\frac{n\Delta L_{Tj}}{I_j+C_s}=\frac{n\sum_{i=0}^{n_f} k_iT_{ij}[\sigma_j^2-(\sigma_j^*)^2]}{\kappa_j\left(\frac{1}{\sigma_j^*}-\frac{1}{\sigma_j}\right)+C_s} \tag{23}$$

The maximum value can be found by taking the partial differential of *ROI* with respect to I_j, setting it equal to zero, solving for $I_{j(\max)}$, and plugging it back into Eq. (23). The explicit solution to this approach is complex and is not given here. Because the function is concave for all values of I_j, the maximum value of *ROI* can also be found using simple solvers.

7 Net Present Value

In many cases, benefits of VR are accrued over a long period. The calculation of benefit needs to take into consideration the effect of time; benefit accrued after investment is made should be discounted by the cost of capital (typically 10–13 percent). The net present value, NPV_V, of an amount V is calculated from r, the periodic interest rate, and n, the period in which the amount is spent (or earned).

$$NPV_V=\frac{V}{(1+r)^n} \tag{24}$$

In the case of VR, calculation of NPV requires a more complex model. Figure 12 shows a graph of expenditures and benefits. The improvement effort starts at T_s and lasts T_I. The investment is spent at a rate R_I ($I_T=R_IT_I$). Once improvement is completed (T_S+T_I), ΔL_T is saved on every product during the remaining life ($T_L-T_S-T_I$). Products are produced at a rate R.

The NPV of the benefit is equal to

$$NPV_B=\sum_{t=0}^{n_P}\frac{\Delta L}{(1+r)^{T_s+T_I+tT_P}} \text{ where } \begin{array}{l} T_P=1/R \\ n_P=R(T_L-T_S-T_I) \end{array} \tag{25}$$

Using a similar calculation, the investment cost, NPV_I can be calculated. The discount cash flow *ROI* (DCF-*ROI*) is the ratio of the two discounted values.

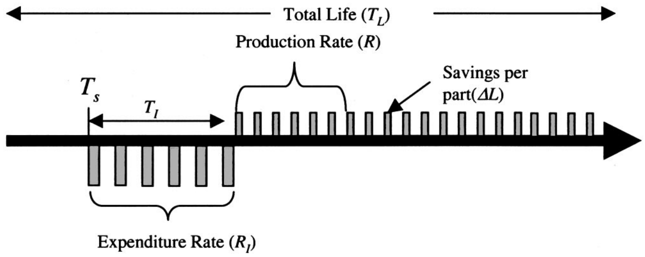

Fig. 12 VR costs and benefits

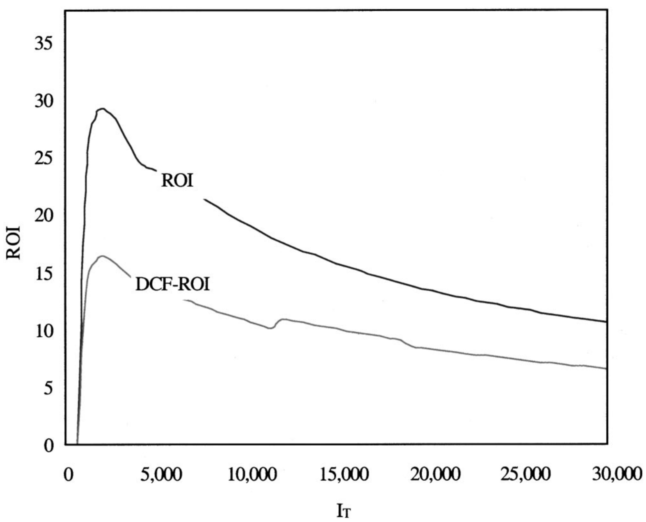

Fig. 13 *ROI* vs. DCF-*ROI*

$$\text{DCF}-ROI=\frac{NPV_B}{NPV_C} \tag{26}$$

Figure 13 shows the differences between the two measures of *ROI*. The regular measure of *ROI* significantly overestimates the value of improvements. Despite these differences, when the distribution was optimized using DCF-*ROI* rather than *ROI* (Eq. (16)), the optimal distributions were virtually identical.

8 Scheduling

In most cases, VR teams are resource limited. The resource rate, R_I, sets the maximum person hours per period dedicated to VR projects. Resources can be applied in a number of ways.[8] For example, the team may decide to work on two projects in parallel or focus on one project at a time. Each has its benefits; scheduling projects in series enables benefits from the first project to become available earlier. However, scheduling projects in parallel moves up the finish time for multiple projects.

Figure 14 shows several scheduling scenarios. In the first case (a), projects are executed serially; in the second case (b), two projects are executed in parallel. T_s for each project is determined by the previous project's completion time. The project duration, T_I, is a function of the expenditure rate (R_I) and the resources (I_j+C_F) needed (assuming C_F is distributed evenly in the project).

The case was run using five scheduling scenarios. It was found that performing the projects in serial always returned a higher DCF-*ROI* (Table 5). It is best to accrue benefit as quickly as possible to offset the time value of money. In addition, because the first project has more impact than the other projects, it was better to finish the first project as quickly as possible and begin to accrue the benefits from the effort.

[8] The problem of project scheduling under constrained resources is the subject of many articles in the operations research field. However, this research takes a simplified view of the scheduling problems because there is no dependency network and there is only a single resource and objective function.

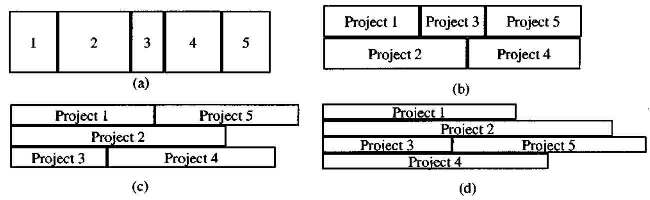

Fig. 14 Scheduling possibilities

Table 5 Number of projects and DCF-*ROI* for I_T=$27,000

Number in parallel	DCF-*ROI*
1	6.93
2	6.60
3	6.25
4	5.92
5	5.59

9 Conclusion

In conclusion, this paper has demonstrated that variation reduction plans can be optimized with minimal computational time. The model requires the design team to create a variation model and estimate both the cost of variation in product-KCs and the cost of variation reduction. Where detailed models are not available, two heuristics can be derived from the analysis. First, projects should be prioritized based on their maximum possible *ROI*. Second, projects should be executed serially. These two heuristics should guide VR teams to execute projects with the largest return first and complete them as quickly as possible.

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Nomenclature

C_F = Fixed cost for variation reduction
C_i = Cost of out-of-tolerance condition for product-KC i
D, **T** = Sensitivity matrices
DCF-*ROI* = Discounted cash flow return on investment
f_i = Nominal value of product-KC i
I_{dj} = Anchor point for variation cost curve
I_j = Cost of improving process-KC j
I_T = Total investment
k = the layer in the KC flowdown
k_i = Taguchi constant for product-KC i
l = Number of layers in the KC flowdown
$[LL_i, UL_i,]$ = Tolerance band for product-KC i
L_i = Taguchi loss for product-KC i
L_T = Total cost of variation
L_T^* = Total cost of variation after improvement
n = Number of product remaining in production life
n_f = Number of product-KCs
n_k = Number of KCs in layer k
n_l = Number of process-KCs

n_l^* = Number of process-KCs subjected to variation reduction
NPV = Net present value
R = Production rate
R_I = Rate of VR expenditure
ROI = Return on investment
T_I = Time to execute VR program
T_L = Total life of product
T_s = Period in which VR is started
x_{jk} = KC j in layer k
Δf_i = Error in product-KC i
Δx_{j2} = Error in KC j
$\Delta \mathbf{x_k}$ = Array of subsystem-KC deviations
$\delta_{\mathbf{k}}$ = Array of partial derivatives
κ_j = Variation cost curve constant
μ_i = Mean of product-KC i
σ_i = Standard deviation of product-KC i
σ_{dj} = Anchor point for variation cost curve
σ_j = Standard deviation of process-KC j (equal to σ_{jnl})
σ_j^* = Target process variation
$\tau_{\mathbf{k}}$ = Array of squared partial derivatives

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