Optimism vs. Pessimism: Design Decisions in the Face of Process Capability Uncertainty

Anna C. Thornton
Assistant Professor,
Mechanical Engineering, MIT
77 Mass Ave. Room 3-449
Cambridge, MA 02139
e-mail: acthornt@mit.edu

Product development firms continually struggle to simultaneously keep product cost low while increasing product quality. This paper focuses on variation and its impact on the quality-failure costs—scrap, rework, extra labor, customer dissatisfaction, and product returns. In many cases, reducing quality failures requires an increase in product cost. Balancing these two costs is challenging because perfect information about process variability is rarely, if ever, available. As a result of process variation uncertainty, there is no clear optimal solution to the quality-failure cost/unit cost tradeoff. The author has observed that companies take one of two approaches to this dilemma: optimistic or pessimistic. The optimistic approach risks high rework costs to ensure the lowest cost product. On the other hand, the pessimistic approach forgoes potential unit cost reductions to avoid any quality failure. This paper presents a utility theory model of decision making under process capability uncertainty. This model is used to describe why either approach can be optimal depending on the organization, market, and product characteristics. In addition, information value theory is applied to explain the relative value of process capability information and variation reduction in the two approaches. The paper uses a frequently encountered design scenario to demonstrate the approach. In addition, a variety of other examples from industry are used to describe how the theory can be applied. [DOI: 10.1115/1.1371774]

Keywords: *Quality, Robust Design, Cost Modeling, Utility Theory, Information Value Theory*

1 Introduction

Product development firms constantly struggle with decreasing product costs while simultaneously improving quality. One approach being used by a variety of companies is to decrease the *quality-failure costs*—scrap, rework, extra labor, customer dissatisfaction, and product returns. Companies are achieving these improvements by employing methods to reduce manufacturing variation and methods to reduce design variation sensitivity.

Two methods can be used to decrease quality-failure costs without increasing product cost. Designers can use parameter design [1], also called robust design, to change a design so it is less sensitive to variation. Alternatively, manufacturers can use variation reduction methods—e.g., statistical process control [2] and Six Sigma methods [3,4]—to reduce manufacturing and process variation.

However, there are circumstances where teams must increase product unit cost to decrease a potential quality-failure cost. For example, a design team may be faced with the question: *is it worth putting more expensive bearings into a machine to reduce the chance of a field failure*? Taguchi [1] calls this process tolerance design. To make this decision, designers must quantify both the unit costs and the expected quality-failure costs. Next, designers must quantify the failure probability [5–7]. However, the validity of the failure mode prediction depends on the existence of perfect information about process standard deviations and mean shifts (also termed process capability).

Perfect information about process capability is rarely, if ever, available [8]. In most cases, process capability can only be accurately expressed as a range and a confidence level (e.g., there is a 95 percent confidence the standard deviation will fall between σ_{ll} and σ_{ul}). Given this uncertainty, designers must still make decisions about configurations, components, and dimensions. If the worst case is assumed, expensive parts, processes, and controls may be needed to offset potential quality failures. This can significantly increase unit cost. If the best case is assumed, the design team can use lower cost parts and processes. However, this approach can result in high quality-failure costs. When faced with this dilemma, a designer must answer: *how much are you willing to increase unit cost to reduce the chance of a quality failure*?

The author has observed two approaches to this dilemma: *optimistic* and *pessimistic*. Optimistic organizations base their design on the best case. They risk high rework costs to ensure the lowest cost product. On the other hand, pessimistic organizations base their designs on the worst case. They forgo potential unit cost reductions to avoid any downstream quality failure. This observation raised several questions.

What is the right decision? This paper proves that both behaviors are rational. Differences in cost competitiveness and volumes are used to explain why organizations take different approaches to process uncertainty.

How can an optimistic approach ever be profitable? The optimistic approach to design, while creating a low-cost product, is inherently very risky. Despite the risk, companies are successfully employing optimistic strategies. It will be shown *that* the success of an optimistic approach is a function of two factors: quickly and effectively reducing both variation and uncertainty. In addition, it will be shown, using information value theory, that process capability data has a higher value in the optimistic case than in the pessimistic case.

How can an organization transition from a pessimistic to an optimistic approach? For example, producers of military systems have traditionally taken a pessimistic approach to design. However, cost pressures in military acquisitions are forcing them to reduce unit costs. One way they can do this is to transition from a pessimistic to an optimistic approach. Suggestions are given on how to effectively make this transfer without sacrificing quality.

Contributed by the Design Theory & Methodology Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received October 1998. Associate Editor: J. Cagan.

2 Existing Work

Works relating to variation and process capability uncertainty fall into three categories: robust design, uncertainty methods, and process capability uncertainty methods.

Robust design is used to optimize product and process parameters to reduce sensitivity to internal and external noise [1]. This can be done experimentally [9] or using computational models [6].

There are a wide variety of approaches to robust design. For example, Parkinson [5] describes a methodology for selecting optimal design solutions where parameters are subject to variation and the design space is bounded by a hard constraint. According to Parkinson, a designer should pick a point, 3σ, away from the constraint. However, this analysis does not include the cost of being away from the optimal.

On the other hand, Yu and Ishii [10] developed methods to match designs with process variability introduced by machining processes. Similar work was done by the same research group [6,11] on injection molding. Soderberg [12] uses the Taguchi loss functions to optimize tolerance design.

All of the above methods focus on reducing the sensitivity of a design to variation. However, they all rely on the existence of perfect information about process capability to determine the optimal design.

Uncertainty models. Uncertainty in decision making is a well-acknowledged problem. There is a variety of literature on decision trees and Bayesian networks. These methods are used to make decisions where there are multiple actions and the outcome is uncertain [13]. Utility theory provides a framework to describe why groups make different decisions under the same uncertainty. According to Arrow [14], decision-making is the process selecting between actions to maximize the utility of the resulting consequences. This method has been applied to a wide variety of fields including design [15,16].

There are many methods to model imprecise information in design. For example, Kim et al. [17] model imprecision by propagating intervals through a constraint set. Finch [18] proposes a method to model imprecise constraints using predicate logic. Otto and Antonsson [19] and Law and Antonsson [20] propose mathematics to propagate imprecise constraints in a design space. Bradley and Agogino [21] use information theory and uncertainty models to assist in component selection. Their system focuses the designer's attention on those factors that are critical and identifies where further information will benefit the search.

Process capability uncertainty. Most process capability uncertainty methods rely on buffers, safety factors, and rules of thumb. For example, Six Sigma methods account for process uncertainty by including a 1.5σ mean shift in the analysis. Another way [22] replaces the standard statistical tolerancing rule, $\sigma = \sqrt{\Sigma \sigma_i^2}$ with $\sigma = (1+\eta)\sqrt{\Sigma \sigma_i^2}$ where η is the safety factor.

In both methods, where there are a large number of contributors to variation, final quality may be significantly underestimated. This can inappropriately increase cost. A method to model process uncertainty has been described by the author [23]. This method propagates uncertainty through a design using a Monte Carlo simulation.

Summary. There are a variety of articles and methods that individually address robust design, decision making under uncertainty, and uncertainty in design decisions. However, there is a distinct lack of research on decision making when there is uncertainty about process capability.

3 Case Study

In order to illustrate optimistic versus pessimistic approaches to uncertainty, a single example will be used throughout the paper. The scenario, positioning two parts in close proximity without touching, appears in a variety of industries. First, the general example will be described, and then a set of industrial examples will be given where this scenario appears. At the end of the paper, more general examples will be given to demonstrate how the results translate to other scenarios.

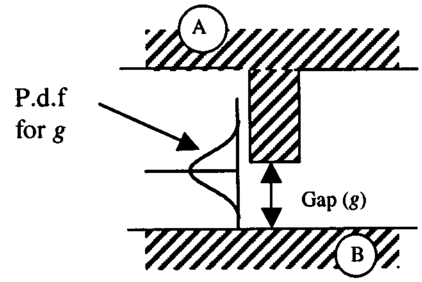

Fig. 1 Specified gap

Two parts, A and B, are located relative to each other using either locating features or a fixture (Fig. 1). The gap, g, between the two parts is ideally zero, because, as the gap increases, part costs increase (i.e., manufacturing time may increase or warranty returns may increase). However, an impact condition—where the gap is less than zero—will necessitate scrap, rework, or repair. Because there is inherent variability in the parts (shown by the Gaussian probability density function), the design team's task is to balance the impact condition probability (i.e., quality-failure cost) against the increased labor content (i.e., increased unit cost).

This design scenario appears frequently in a variety of industries.

- **Aircraft**. Aircraft structures cannot be significantly deflected during assembly because built-in stresses can reduce aircraft fatigue life (usually a maximum pull-up force is specified). To avoid stressing parts, a combination of designed-in gaps and shims are used to mate parts. When an impact condition occurs, parts may need to be reworked, trimmed, or, in the worst case, scrapped.
- **Automotive body panels**. Gaps between automotive body panels are a key quality characteristic [24]. Gaps around the door, hood, and trunk are highly visible to the customer: the larger the gap (or worse, the more uneven the gap), the lower the perceived quality. However, there are two problems with a small gap. When body panels overlap, significant rework is required. Second, variation in a narrow seam is more visible than in a wide seam.
- **Automotive components**. One key quality characteristic of many automotive drive train components is durability. Durability is often a function of how closely parts fit together [25]. However, if the gap is too small, parts may bind and not function.
- **Ships**. In the ship building industry, welding is used to fill gaps between structural elements. Larger gaps take longer to fill and increase both the labor content and the cycle time. In some specialized applications, additional filler material also adds cost. However, if there is an impact condition between components, parts must be separated and trimmed, resulting in large rework costs.

All of these cases are characterized by an uneven cost/loss function: i.e., the gap, g, is subject to both a *larger is better* and a *smaller is better* condition [1]. The larger the gap, the less likely an impact condition will occur; however, the smaller the gap, the lower the production costs. A nominal gap, g, is specified to balance the unit cost (smaller is better) against a quality-failure cost (larger is better). If the gap is too small, higher than acceptable quality failures may occur. If the gap is too large, the product unit cost will be too high. The problem of selecting the optimal mean has been described by a number of authors [26].

4 Variation Cost

Quantification of variation cost begins with a definition of a cost/loss function. This function describes the cost incurred when the gap, as produced, has a value of g_p (Fig. 2). In this case, the

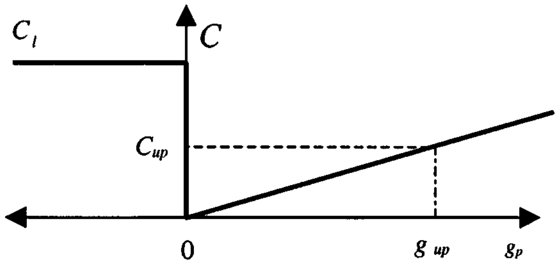

Fig. 2 Cost curves

left-hand side (an impact condition) is modeled as a step function ($C = C_l$). When a part is trimmed there is no cost difference between trimming one inch or two inches. The right hand side (positive gap) is modeled using a linear function. For example, weld time and volume increases linearly with an increase in the gaps between ship panels. A single characteristic point anchors this curve (g_{up}, C_{up}).[1] Equation 1 describes the cost/loss function.

$$C(g_p) = \begin{cases} C_l & \text{if } g_p < 0 \\ \dfrac{C_{up}}{g_{up}} g_p & \text{if } g_p > 0 \end{cases} \tag{1}$$

Assuming no variation, the optimal gap is zero. However, the gap will vary according to a probability density function (p.d.f), $p(g_p)$. Variation in g_p can be modeled as a Gaussian distribution[2] with a mean of g, and a standard deviation of σ. The expected cost is a function of $P(g_p)$ and $C(g_p)$.

$$C_{\text{exp}}(g,\sigma) = \underset{[p_g]}{E}[C] = \int_{-\infty}^{\infty} C(g_p) p(g_p) dg_p \tag{2}$$

After integrating and removing terms whose contribution is minimal, the expression for $C_{exp}(g,\sigma)$ is derived where erf is *the* error function.

$$C_{\text{exp}}(g,\sigma) = \frac{C_l}{2}\left[erf\left(\frac{-g}{\sigma\sqrt{2}}\right) + 1 \right] + \frac{C_{up}}{g_{up}} \frac{8}{2} \left[1 + erf\left(\frac{g}{\sigma\sqrt{2}}\right) \right] \tag{3}$$

The optimal gap, $g_{min}(\sigma)$, satisfies min($C_{exp}(g,\sigma)$). Equation 3 is graphed in Fig. 3 for four values of σ (0.00, 0.01, 0.02, and 0.03). The curves *were* calculated using $C_l = 100$, $C_{up} = 5$, and $g_{up} = 0.03$. The relative values of these numbers reflect examples seen in industry.

Figure 3 also demonstrates how the cost is locked in by design. The minimum unit cost for any gap is defined by the bottom curve ($\sigma = 0$).

$$C_{\text{exp}}(g,0) = g \frac{C_{up}}{g_{up}} \tag{4}$$

If the standard deviation is assumed to be 0.03, minimizing C_{exp} results in a gap of 0.06. In this case, even if the variation is reduced to zero, the product cost cannot be reduced below 10. However, if the standard deviation is assumed to be 0.01, minimizing C_{exp} results in a gap of 0.02. In this case, if the variation is reduced to zero, the cost could be theoretically reduced as low as 5.

[1]This curve can also be modeled as a Taguchi quadratic function. The conclusions of the paper are not altered when the alternative function is used. The quadratic function tends to estimate a higher net cost.

[2]Other distributions will alter the equations and the numerical results. However, the overall conclusions of the paper are not affected by the shape of the probability density functions.

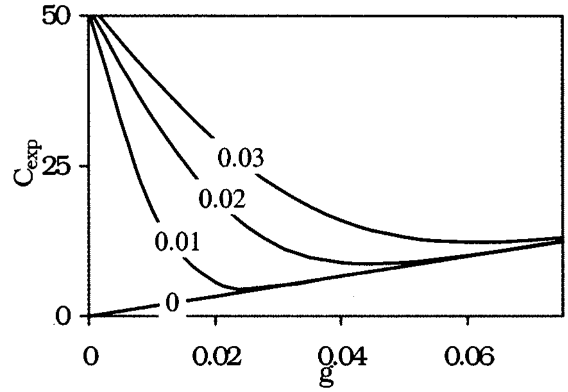

Fig. 3 C_{exp} vs. g for four values of σ (0.0, 0.010, 0.020, and 0.030)

5 Uncertainty Model

Equation 3 enables a design team to calculate the optimal gap. However organizations are rightly wary of such analyses because ''the process capability may be worse than we expected.'' This is a valid concern because processes degrade over time, suppliers change, and unexpected problems occur. As a result, the true optimal value for g is not known a priori. However, designers are required to set the gap in the design stage before the final process capability is known. This section addresses the question posed at the beginning of the paper: *what is the right decision*?

5.1 Sources of Uncertainty. There are two methods used to predict process capability data for internal parts: process capability databases and manufacturing knowledge. Process capability databases provide surrogate data for similar parts [27,28]. However, surrogate data is not always an accurate performance indicator for a new design. Differences in material, geometry, and process parameters can result in different standard deviations and mean shifts. Where there are multiple surrogate parts there may be multiple standard deviation values. In addition, inaccuracies arise because process capability measures are often based on short-term process performance and may not include process degradation due to tool wear, lack of machine maintenance, or operator effects. Where process capability databases do not exist, manufacturing functions are required to make educated guesses about variation. Guesses are based on experience and are often communicated as a range rather than a specific value.

Outsourced parts have an even higher uncertainty. In some cases, suppliers are unable or unwilling to share their capability. For others, the supplier may not have been selected and the designer must design for a range of potential suppliers.

5.2 Process Capability Uncertainty. To capture uncertainty, process capability can be expressed as a range $[\sigma_{ll}, \sigma_{ul}]$, where there is a 95 percent probability the standard deviation will fall in the range (corresponding to $2\sigma_\sigma$). To enable uncertainty analysis, the range is translated into a probability distribution. Process capability can be modeled as a Gaussian distribution with a mean, μ_σ, and a standard deviation, σ_σ, where

$$\mu_\sigma = \frac{\sigma_{ul} + \sigma_{ll}}{2}, \quad \sigma_\sigma = \frac{\sigma_{ul} - \sigma_{ll}}{4} \tag{5}$$

Figure 4 demonstrates why decision making under process capability uncertainty is difficult. The bottom curve ($\sigma_{ll} = 0.010$) represents the minimum unit cost assuming variation cannot be cost reduced below σ_{ll}. The top curve ($\sigma_{ul} = 0.020$) shows the curve for the worst-case scenario and represents the highest possible value of C_{exp}. There is an optimal mean corresponding to each case, g_{ll} and g_{ul}. These two values bound potential values for g.

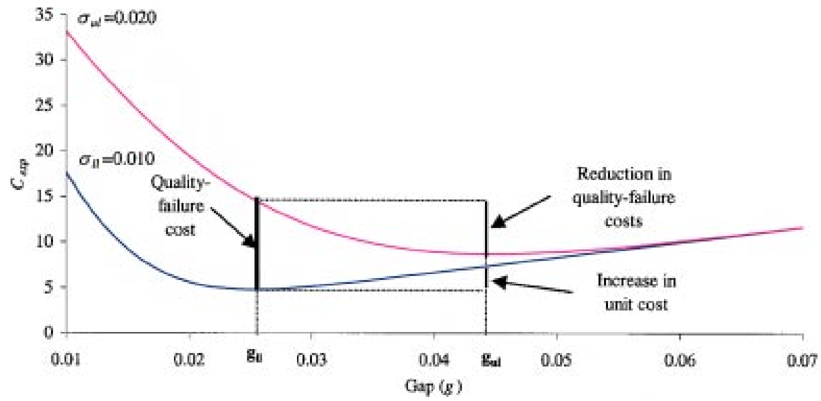

Fig. 4 Worst and best cases for process capability

If σ is assumed to be σ_{ll}, the optimal gap, g_{ll}, is 0.025 resulting in a C_{exp} of approximately 4.8. However, if the σ turns out to have a value of σ_{ul}, the cost will be 14.6 (a three-fold increase). This delta is labeled *quality-failure cost.* The quality-failure cost is the amount of additional cost created by the higher than expected variation. By increasing *g*, unit cost is increased. This increase is labeled *increase in unit cost*. However increasing *g* simultaneously reduces quality-failure cost *because larger gaps are less sensitive to variation*. This decrease is labeled *reduction in quality-failure costs.*

5.3 Expected Cost Under Uncertainty. One way to determine optimal gap is to minimize the expected value under uncertainty. $C_{uc}(g,\sigma_\sigma,\mu_\sigma)$ is calculated by

$$C_{uc}(g,\sigma_\sigma,\mu_\sigma)=\underset{p_\sigma}{E}[C_{\exp}(g,\sigma)]$$
$$=\int_{-\infty}^{\infty} C_{\exp}(g,\sigma)\frac{1}{\sigma_\sigma\sqrt{2\pi}}e^{-(\sigma-\mu_\sigma)^2/2\sigma_\sigma^2}d\sigma \tag{6}$$

The optimal gap satisfies the expression min $C_{uc}(g,\sigma_\sigma,\mu_\sigma)$ (*in this case* $g\approx 0.037$). However, it was observed that designers rarely choose this value, rather they tend to design to one extreme or the other. The next section will explain, using utility theory, why designers tend to choose either the g_{ul} or g_{ll}.

6 Optimism vs. Pessimism

The author has observed organizations taking very different strategies when faced with the same uncertainty. Some companies take a *pessimistic* approach to uncertainty and always design for the highest variability, g_{ul}. Other organizations take an *optimistic* approach and design to the lowest expected variation, g_{ll}. The rest of this section describes why both approaches are rational depending how companies value unit cost relative to potential quality failures. First, observations are made about the correlation between the two approaches and organization characteristics. Second, using utility theory this correlation will be shown to be rational.

6.1 Optimistic vs. Pessimistic. These observations are based on a large number of site visits, interviews, and extended visits at over thirty companies.

Optimistic Approach. High-volume and/or low-margin organizations tend to exhibit optimistic behavior when faced with process capability uncertainty. Companies taking an optimistic approach include automotive body assembly and chip manufacturers. For example, automotive industries tend to set tighter gaps and gap tolerances than their current process capability. These targets are selected to place themselves competitively against other organizations [24]. Chip manufacturers continue to reduce their line widths below their current capability to reduce both unit cost and increase performance. Optimistic organizations also tend to use production prototyping and product ramp cycles to identify and remove variation [29].

Pessimistic Approach. On the other hand, it has been observed that companies who produce high-price and/or low-volume products tend to take the pessimistic approach. Historically, these companies have competed on product quality and performance rather than on price. For example, many military contractors operate under cost-plus contracts. Under these contracts, the price is based on the final cost to manufacture plus a percentage. There is very little incentive to reduce unit cost but a high incentive to design the highest performance product possible. Pessimistic approaches are also often used where there are limited production prototypes.

6.2 Utility Theory. Differences between optimistic and pessimistic approaches can be ascribed to differences in the utilities of unit cost and quality-failure costs. As stated above in Section 2, utility theory uses a measure of outcome utility (i.e., value) to rank preferences. The optimal action maximizes the outcome utility.

In this case, selecting *g* is the action. There are two outcomes for every value of *g*: a unit cost, $C_u(g)$, and a potential quality-failure cost, $C_q(g)$. The unit cost is defined by the lower curve in Fig. 4. It represents the lowest possible expected cost for a given gap, *g*.

$$C_u(g)=C_{\exp}(g,\sigma_{ll}) \tag{7}$$

ΔC_u is the difference between maximum unit cost and unit cost at gap, *g*.

$$\Delta C_u(g)=C_u(g_{ul})-C_u(g) \tag{8}$$

The quality-failure cost is defined by difference between the upper and lower curves in Fig. 4. It represents the increase in price due to increased variation for a given gap, *g*.

$$C_q(g)=C_{\exp}(g,\sigma_{ul})-C_{\exp}(g,\sigma_g) \tag{9}$$

ΔC_q is the difference between the maximum quality-failure cost and the quality-failure cost at gap *g*.

$$\Delta C_q(g)=C_q(g_{ll})-C^q(g) \tag{10}$$

The utility function is based on the question posed in Section 1: *how much are you willing to increase unit cost to reduce the chance of a quality failure later*? This question can be translated into utility theory terminology: *what are the relative utilities of decreasing unit costs versus decreasing quality-failure costs*?

Table 1 Utility of unit vs. quality-failure costs

	U_u	U_q
Value unit cost	1	0
Value quality failure	0	1

Table 2 Actions and outcomes

		Outcomes	
		ΔC_u	ΔC_q
Actions	*Optimistic* $(g=g_{ll})$	$C_u(g_{ul})-C_u(g_{ll})$	$C_q(g_{ll})-C_q(g_{ll})=0$
	Pessimistic $(g=g_{ul})$	$C_u(g_{ul})-C_u(g_{ul})=0$	$C_q(g_{ll})-C_q(g_{ul})$

Table 3 Optimal scenario for each strategy

Utility		*Actions*		*Optimal decision*
		Optimistic	Pessimistic	
Values	*Value unit cost*	$C_u(g_{ul})-C_u(g_{ll})$	*0*	***Optimistic***
	Value quality-failure cost	*0*	$C_q(g_{ll})-C_q(g_{ul})$	***Pessimistic***

$U(g)$ is the total utility for a gap g; U_u is the utility of a decrease in the unit cost ($\Delta C_u(g)$); and U_q is the utility of a decrease in a quality-failure cost ($\Delta C_q(g)$).

$$U(g)=U_u\Delta C_u(g)+U_q\Delta C_q(g) \tag{11}$$

Utilities. There is a continuum of possible values for U_q and U_u. To simplify the analysis, only two bounding conditions are analyzed. In the first case (value unit cost), the organization puts a high utility on the minimum product cost and does not value reducing potential quality-failure costs. In the second case, there are the organizations that place a high value on potential failure costs but don't value minimizing unit cost (value quality failure). Table 1 shows the utility values for the two cases.

Actions and outcome. Similarly, two actions, optimistic and pessimistic, bound the continuum of actions. Table 2 shows the two outcomes for each action.

Relative utility of outcomes. Next, the relative utilities of the outcomes for each action are calculated. This is done by multiplying the cells in Table 1 and Table 2 using Eq. (11).

Table 3 shows the two bounding value systems and the two bounding actions. The cells in the table show the relative utilities for each action for each value system. The optimal decision maximizes the utility. The results show that valuing unit cost reductions is consistent with taking an optimistic approach. On the other hand, those companies who value reducing potential quality failures should take a pessimistic approach.

7 Variation and Uncertainty Reduction: The Missing Link

Table 4 shows the values of C_u, C_q, and C_{uc} for the two approaches. If unit costs and probabilistic quality-failure costs are considered static, the optimistic case will most likely result in a higher net cost (as defined by C_{uc}). However, companies still take the optimistic approach to design. For example, chip manufacturers like Intel and AMD continue to reduce their line widths even though their initial yields are very low. This section addresses the second question posed in the introduction: *how can an optimistic approach ever be profitable*?

Table 4 Costs associated with each approach

	C_u	C_q	C_{uc}
Optimistic Approach	4.8	14.6	8.9
Pessimistic Approach	7.5	8.8	7.7

7.1 Variation Reduction. This apparent contradiction can be explained by taking a dynamic rather than a static view of process capability. If process capability is viewed as static—not subject to improvement—the optimistic case will, on average, be less cost effective. However, process capability is not static. Process variation reduction has been the subject of many books, articles, and company initiatives. For example, Six Sigma is being successfully applied by GE, AlliedSignal, and Motorola to reduce variation in existing manufacturing processes.

In addition, Intel's success relies on actively pursuing increasing yields [30]. Rapidly increasing yields gives them a significant edge over their competitor, Advanced Micro Devices (AMD). AMD struggled to increase yields on their chips and their inability to quickly improve yields impacted their profitability and competitiveness [31].

The competitive advantage enabled by variation reduction and continuous quality improvements has also been described in a number of articles and books. For example, Fine [32] provides a quantitative proof of the benefits of continual quality improvements. The benefits of continual improvement and learning have also been described in detail by Clark and Fujimoto [33] and Womack, et al. [34].

The effectiveness of variation reduction is also a function of the production rates. The variation reduction cycle includes baselining the process, running experiments, identifying the largest contributors, imposing control on those contributors, and confirming the variation reduction. When companies produce at low rates, variation reduction can take a large percentage of the total production run to execute. High-volume producers have more opportunity to reduce variation and the benefits are amortized over more products.

7.2 Uncertainty Reduction. When the optimistic approach is taken, it assumes organizations are capable of reducing variation to the best case. However, inaccurate predictions may underestimate σ_{ll}. In this scenario, net cost may be higher than originally expected. Therefore, it is in the best interest of an organization to make accurate predictions about process capability. Leland [26] describes the effect of underestimating target process variation. The case subject used significant resources during ramp-up trying to remove a variation problem. Because they were unable to reduce variation to a satisfactory level, the final solution included an expensive design change.

Process capability uncertainty can be reduced using any number of methodologies: prototypes, process models, and cross-functional interaction with suppliers and manufacturers. However, uncertainty reduction requires resources. To determine how much uncertainty should be reduced, both the cost and benefit of information must be quantified.

Uncertainty reduction. First, uncertainty reduction must be quantified. Reduction in uncertainty is described by Rothschild and Stiglitz [35] as a reduction in the variance from σ_σ to σ'_σ around a static mean.

$$R=\frac{\sigma'_\sigma}{\sigma_\sigma} \tag{12}$$

However, this measure has one major shortcoming: it does not quantify the resultant mean, μ_σ. For example, if the original bounds range from 0.010 to 0.050 and uncertainty is reduced by 50 percent (i.e., $R=0.5$), the resulting mean can vary between 0.020 at best and 0.040 at worst (Fig. 5). Because it is not possible to predict the resulting mean, it is necessary to average across a range of potential values.

Information benefit. Information benefit is typically measured as a reduction in the expected cost [36]. In this case, the expected cost is C_{uc} described in Eq. 6. The expected cost after uncertainty reduction, C'_{uc} is:

$$C'_{uc}=1/2C_{uc}(g,\sigma'_\sigma,\mu'_\sigma)+1/2C_{uc}(g,\sigma'_\sigma,\mu''_\sigma) \tag{13}$$

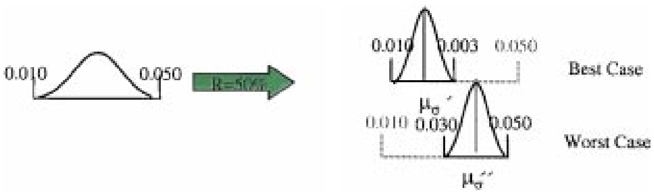

Fig. 5 Uncertainty reduction

where σ'_σ is the reduced uncertainty and μ'_σ and μ''_σ are the upper and lower bound on the mean. The benefit of information is the difference in the expected cost before and after information.

$$\Delta C_{uc} = C_{uc} - C'_{uc} \tag{14}$$

Cost of information. The next step is to quantify information cost. The measure of information content/entropy of a system, H, was proposed by Shannon and Weaver [36]. Large values of H correspond to high entropy and low information quality. Changes in H have been used as an information cost measure by a variety of researchers [14,37]. In most cases, the cost of uncertainty reduction is assumed to be linear with ΔH.

The information entropy, H, [38] for a problem with n possible outcomes where each outcome has a probability p_i is:

$$H = -K \sum_i p_i \log p_i \quad \text{where} \quad \sum_i p_i = 1 \tag{15}$$

K is a constant used to scale H. H is continuous with p_i, is at a maximum where all p_i are equal, and has a minimum value of zero when one value of p_i is one. The analogous case for a continuous distribution with a probability distribution function $p(x)$ is:

$$H = -K \int_{-\infty}^{\infty} p(x) \log p(x) dx \tag{16}$$

In this case, $p(\sigma)$ is the p.d.f of σ and H is

$$H = -K \int_{-\infty}^{\infty} p(\sigma) \log p(\sigma) d\sigma = K \log(\sigma_\sigma \sqrt{2e\pi}) + K \frac{\mu_\sigma^2}{2\sigma_\sigma^2} \tag{17}$$

Shannon and Weaver [36] note one important difference between the continuous and discrete entropy measures. The discrete entropy is strictly bounded and is always positive. The continuous case, however, is a relative measure and cannot be guaranteed to always be positive. The scaling problem is avoided by measuring information entropy deltas rather than an absolute value. In this case, the change in information entropy, ΔH, due to a reduction in uncertainty from σ_σ to σ'_σ is[3]

$$\Delta H = -K \log(R) \quad \text{where} \quad R = \frac{\sigma'_\sigma}{\sigma_\sigma} \tag{18}$$

Figure 6 plots cost against information benefit for both the optimistic and pessimistic case. ΔC_{uc} is the benefit (Eq. (14)) and $-\log R$ is the information cost (Eq. (18)). The figure shows that in both cases, there is a decreasing marginal return for additional information. In addition, it shows there is a dramatically larger benefit of information in the optimistic case than the pessimistic case.

The optimistic case benefits more from information because the potential quality-failure costs are significantly higher. The optimistic approach can be viewed as the riskier option although the benefits are higher. In general, the riskier a situation, the more information benefits the decision-maker [14].

[3]This assumes the second term in Eq. (17) is a normalization term that scales the entropy. It is therefore not included in the analysis.

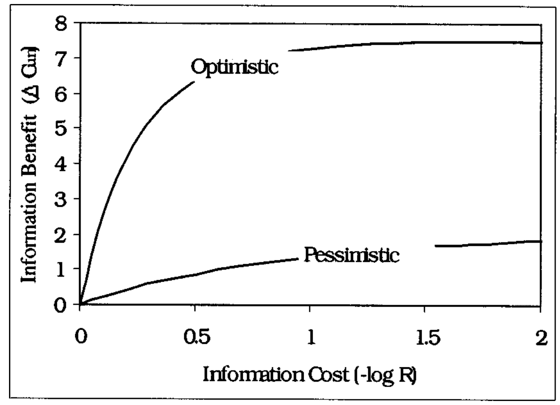

Fig. 6 Cost vs. benefit of information

The benefit of information in the optimistic case is further enhanced by the low-volume/high-volume differences discussed in Section 6.1. The net information benefit, B, is the difference between the benefit of information and the cost of the information. B is a function of the cost (Eq. (14)) and benefit of information (Eq. (18)) as well as the remaining production volume, V.

$$B = V \Delta C_{un} - K(-\log R) \tag{19}$$

As stated above, most optimistic companies tend to be high volume producers. The higher the volumes, the more benefit is derived from the information gathered because the investment is amortized over more products.

Ertan [38] surveyed a variety of companies about their variation reduction methods. The surveyed companies were split into two groups: commercial/high-volume companies and military/low-volume companies. The survey showed high-volume producers were significantly better at capturing process capability data than low-volume producers. Process capability data is used to reduce uncertainty about the final variation. The better the data, the greater the certainty. This difference may be explained by the findings in this paper. First, low volume producers are less able to collect process capability information because of the lack of datapoints. The second, suggested by this analysis, is that there is a greater incentive to understand process capability where an optimistic approach is used.

8 Industry Trends

This section addresses the third question posed at the beginning of the paper: *how can an organization transition from a pessimistic approach to an optimistic approach*?

As the market requires companies to become more cost competitive, they are being forced to reduce their unit cost. For example, historically Boeing's success has been based on performance rather than cost; however, they now are being pushed to reduce cost. As an article in *Forbes* [39] stated ''performance, delivery, and technology mattered more than cost ... but airliner design has plateaued, and the airlines that buy the planes today are chiefly interested in how much carrying capacity they can buy for a buck.''

Pushing process capability limits and taking a more optimistic approach should decrease unit costs. For example, military producers are being forced to become more cost competitive. They are being required to include off-the-shelf components rather than military specification parts (milspec). The off-the-shelf components are lower cost but there is generally less certainty about their quality and performance.

However, rapid shifts to optimistic from pessimistic approaches can be dangerous and expensive. As shown in Section 7, without lowering uncertainty and proactively reducing variation in production, there is a large potential downside to adopting optimistic approaches.

How can companies who have traditionally taken a pessimistic approach transition without encountering massive and expensive quality problems? For high-volume companies this is typically done through SPC, production prototypes, and proactive design of variation reduction plans. However, in the case of low-volume companies, the application of the optimistic approach is trickier.

There are two barriers to transitioning from a pessimistic approach to an optimistic approach in low-volume companies. First, low volumes hamper uncertainty reduction. The amount of data available from SPC and other data collections will be limited. Learning is also limited by the lack of a process ramp—the first product built is usually sold and there are limited production runs.

This paper suggests three methods to overcome these barriers. First, organizations should make a slow transition from pessimistic to optimistic strategies. Transitions should be made first where the cost impact is low and where data collection can be leveraged across other parts. Second, extensive modeling and simulation should be used. Because production prototypes can be prohibitively expensive, tools such as Variation Systems Analysis (VSA) [40] can be used to simulate assembly.[4] By learning through simulation, high-risk assemblies can be identified prior to production. Variation reduction plans can be initiated at the start of production rather than waiting for quality problems to occur. Third, uncertainty reduction should leverage surrogate processes, similar parts, common parts, and supplier parts. By sharing learning over a larger number of similar parts, the quality of information can be improved.

In summary, low volume products *can* take a more optimistic approach to variation. However, a high degree of planning and analysis is required because they do not have the luxury of production prototypes and high production volumes. In addition, there may be some sub-systems where, given the high degree of uncertainty, the optimal approach may never be feasible.

9 Other Examples

These conclusions do not only apply to the gap example but can be generalized to other decisions about design configurations and manufacturing processes. Three examples are used to demonstrate this generalization: electronic assembly, adjustment parameters, and aircraft assembly.

9.1 Computer Assembly. The optimistic/pessimistic dichotomy does not necessarily apply to entire organizations. Different product platforms (even products in the same product family) may require different approaches to uncertainty. For example, traditional computer assembly used screws to make connections between sub-systems. However, the use of screws increases assembly time. To reduce labor content, many companies now use snap fit connectors. Snap fits reduce assembly costs but have a higher rate of connection failure. Variation in the moldings can cause faulty connections to occur. The optimistic approach is to use snap fits; the pessimistic is to use screws.

Depending on the product characteristics, either the optimistic or pessimistic approach may be appropriate. For example, as the market size for the low-cost computer continues to grow, the pressure to produce lower cost computers increases [41,42]. In this market segment it is in the best interest of the organization to take a very optimistic approach to the product design. In order to be cost competitive, snap fits are probably more appropriate than the use of screws.

However, there is another class of computers—the high end/high performance systems where reliability is a dominant competitive factor. In this case, avoiding product returns and ensuring reliability may be worth the extra cost. Taking a pessimistic approach is probably the appropriate approach.

[4]There is always the risk, however, that the cost of simulation and modeling can be quite high.

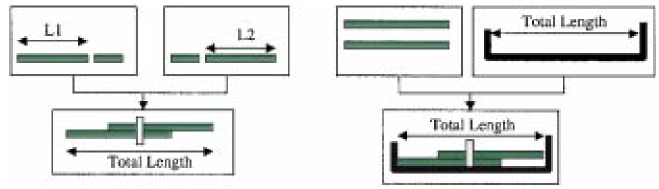

Fig. 7 Determinate and fixtured assembly

9.2 Adjustment Parameters. The least expensive manufacturing process requires no special tools, steps, labor, or tuning. Parts are picked from bins and assembled without special adjustments or fitting. Pick-and-place reduces the error rate, decreases labor content, and removes the need for specialized training. However, variation can hamper the efficiency of pick-and-place assembly. When too much variation exists, parts may not fit or function properly creating rework and production disruptions.

To compensate for variation, design teams may decide to include adjustments or tuning parameters to prevent quality failures [43]. For example, a voltage might be tuned in the factory to achieve proper copy print quality. Although adjustment processes ensure product acceptability, they increase the unit product cost because additional equipment, training, and labor are required. In addition, adjustments have been correlated with field problems.

Many companies are trying to reduce their dependence on adjustment parameters. However, this transition should not be made without understanding the effects of uncertainty, proactively planning variation reduction, and increasing process capability data collection.

9.3 Aircraft Assembly. Historically, aircraft frame assembly has relied on specialized tooling and processes to account for high part variability (i.e., fixtured assembly). However, as cost pressures increase, aircraft companies are adopting assembly, methods similar those used by automotive assembly (i.e., determinate assembly) [7]. Figure 7 shows the differences between the two assembly methods. In determinate assembly (left figure), each panel has predrilled location holes. Assembly involves lining up the holes and inserting a fastener. The total length depends on the precision of the hole location. Fixtured assembly (right figure) uses a dimensional fixture to define the total length. In this case, two blank panels are placed in the fixture and a hole is drilled simultaneously through both. After match drilling, the parts must be pulled apart, the burrs removed, and the parts realigned. Fasteners are then inserted in drilled hole.

Determinate assembly is less expensive because it removes the need for specialized fixtures and also reduces the labor content. However, determinate assembly is highly sensitive to hole location variation and is at higher risk of quality failures. In addition, the failures are very expensive; if a part does not fit, the part may have to be scrapped. The fixtured approach is more expensive on a unit cost basis because it requires additional labor, expensive dedicated fixtures, and prevents interchangeability and replaceability of parts. However, it is less sensitive to part variation.

10 Summary

This paper contributes several new ideas. First, it proposes a formalism for selecting the optimal design under process capability uncertainty. Second, it proposes a quantitative explanation for various behaviors under uncertainty. Third, it describes why learning and variation reduction are critical in cost competitive industries. Fourth, it explains why it is critical for companies, when making a shift from a pessimistic to an optimistic approach to variation, to simultaneously implement methods to reduce uncertainty as well as reduce variation.

Acknowledgments

The author would like to acknowledge the support provided by NSF Career Grant #9702662 managed through the Division of Design, Manufacturing & Industrial Innovation in the Engineering Directorate, the Center for Innovation in Product Development under NSF Cooperative Agreement Number EEC-9529140, and the Lean Aerospace Initiative. Specifically, the author wishes to acknowledge the input and feedback from William J. Brougham.

Nomenclature

B = Net benefit of information
$C(g_p)$ = Cost when gap of size g_p is produced
$C_{\exp}(g,\sigma)$ = Expected cost of a gap, g, for a given standard deviation, σ
C_I = Impact cost
C_q = Quality-failure cost
C_{up} = Cost at g_{up}
C_u = Unit cost
$C_{uc}(g,\sigma_\sigma,\mu_\sigma)$ = Expected cost under uncertainty
ΔC_q = Difference between maximum unit cost and unit cost at gap, g
ΔC_μ = Difference between the maximum quality-failure cost and the quality-failure cost at gap g
g = Designed gap
g_p = Gap as produced
$g_{\min}(\sigma)$ = Gap that minimizes $C_{\exp}$ for a given standard deviation, σ
g_{ul} = $g_{\min}(\sigma_{ul})$
g_{ll} = $g_{\min}(\sigma_{ll})$
g_{up} = Anchoring point for cost/loss curve, $C(g_p)$
H = Information entropy
ΔH = Decrease in information entropy
η = Process capability safety factor
U_q = Utility of a quality-failure cost decrease
U_u = Utility of a unit cost decrease
$U(g)$ = Total utility for gap (g)
$p(g_p)$ = Gap probability density function
σ, σ_i = Standard deviation
σ_{ll} = Lower level of potential process standard deviation
σ_{ul} = Upper level of potential process deviation
σ_σ = Standard deviation of process capability distribution
σ'_σ = Standard deviation after uncertainty reduction
μ_σ = Mean value of process capability distribution
μ'_σ = Lower bound on mean value after uncertainty reduction
μ''_σ = Upper bound on mean value after uncertainty reduction
R = Uncertainty reduction
K = Scaling factor for information cost
V = Volume

References

[1] Taguchi, G., 1993, *Taguchi on Robust Technology Development: Bringing Quality Engineering Upstream*, ASME Press, New York.
[2] Montgomery, D. C., 1996, *Introduction to Statistical Quality Control*, John Wiley & Sons.
[3] Clifford, H., 1997, ''Six Sigma,'' *Continental*, November, pp. 64–67.
[4] Doherty, J., 1997, ''Mixed Signals: After a Search, AlliedSignal's Chief Aims to Win the Street's Favor Again,'' *Barrons*, December 19, pp. 29–34.
[5] Parkinson, A., 1995, ''Robust Mechanical Design Using Engineering Models,'' ASME J. Mech. Des., **117B**, pp. 48–54.
[6] Kazmer, D., Barkan, P., and Ishii, K., 1996, ''Quantifying Design and Manufacturing Robustness Through Stochastic Optimization Techniques,'' Design Automation Conference, ASME Design Technical Conferences, Irvine, CA.
[7] Thornton, A. C., 1997, ''Using Key Characteristics to Balance Cost and Quality During Product Development,'' *Design Theory and Methodology Conference, ASME Design Technical Conferences*, Sacramento, CA.
[8] Tata, M., and Thornton, A., 1999, ''Process Capability Database Usage In Industry: Myth vs. Reality,'' *Design for Manufacturing Conference*, ASME Design Technical Conferences, Las Vegas, NV.
[9] Phadke, M. S., 1989, *Quality Engineering Using Robust Design*, PTR Prentice-Hall Inc, Englewood Cliffs, New Jersey.
[10] Yu, J.-C., and Ishii, K., 1994, ''Robust Design by Matching the Design with Manufacturing Variation Patterns,'' *Design Theory and Methodology, ASME Design Engineering Technical Conferences*, Minneapolis, MN, pp. 7–14.
[11] Beiter, K. A., and Ishii, K., 1996, ''Incorporating Dimensional Requirements into Material Selection and Design of Injection Molded Parts,'' *Computers in Engineering Conference*, 1996 ASME Design Engineering Technical Conferences, Irvine, CA.
[12] Soderberg, R., 1994, ''Robust Design by Tolerance Allocation Considering Quality and Manufacturing Costs,'' *Design Theory and Methodology*, ASME Design Engineering Technical Conferences, Minneapolis, MN, New York, pp. 219–225.
[13] Marshall, K. T., and Oliver, R. M., 1995, *Decision Making and Forecasting*, McGraw Hill Inc., New York.
[14] Arrow, K. J., 1971, *Essays in the Theory of Risk-Bearing*, Markham Publishing Company, Chicago.
[15] Hazelrigg, G. A., 1998, ''A Framework for Decision-Based Engineering Design,'' ASME J. Mech. Des., **120**, pp. 653–658.
[16] Thurston, D. L., and Locascio, A., 1994, ''Decision Theory for Design Economics,'' Eng. Econom., **40**, No. 1, pp. 41–72.
[17] Kim, K., Cormier, D. R., O'Grady, P. J., and Young, R. E., 1995, ''A System for Design and Concurrent Engineering Under Imprecision,'' J Intelligent Manufacturing, **6**, No. 1, pp. 11–27.
[18] Finch, W. W., 1998, ''An Overview of Inference Mechanisms for Quantified Relations,'' *Design Theory and Methodology*, ASME Design Engineering Technical Conferences, Atlanta, GA, pp. DETC98/DTM-5666.
[19] Otto, K. N., and Antonsson, E. K., 1995, ''Propagating Imprecise Engineering Constraints,'' *Proceedings of the 1995 FUZZ/IEEE Conference*, Yokohama, JP.
[20] Law, W. S., and Antonsson, E. K., 1994, ''Including Imprecision in Engineering Design Calculations,'' *Design Theory and Methodology Conference*, ASME Design Technical Conferences, Minneapolis, MN.
[21] Bradley, S. R., and Agogino, A. M., 1994, ''An Intelligent Real Time Design Methodology for Component Selection: An Approach to Managing Uncertainty,'' ASME J. Mech. Des., **116**, pp. 980–988.
[22] Liggett, J. Y., 1992, *Dimensional Variation Management Handbook: A Guide for Quality, Design, and Manufacturing Engineers*, Prentice Hall.
[23] Thornton, A. C., 1999, ''Variation Risk Management Using Modeling and Simulation,'' ASME J. Mech. Des., **121**, No. 2, pp. 297–304.
[24] Leland, C., 1997, ''*A Cultural Analysis of Key Characteristic Selection and Team Problem Solving During an Automobile Launch*,'' S.M., MIT, Boston.
[25] Lee, D., and Thornton, A., 1996, ''The Identification and Use of Key Characteristics in the Product Development Process,'' *Design Theory and Methodology Conference*, ASME Design Technical Conferences, Irvine, CA.
[26] Tang, K., and Lo, J.-J., 1993, ''Determination of the Optimal Process Mean when Inspection is Based on a Correlated Variable,'' IIE Transactions, **25**, No. 3, pp. 66–72.
[27] Lucca, A. M., Berti, K. N., and Cerveny, D. C., 1995, ''Statistical Tolerance Allocation in Design Utilizing Historical Supplier Process Data,'' *Advances in Electronic Packaging*, ASME pp. 229–235.
[28] Naish, J. C., 1996, ''Process Capability Modeling in an Integrated Concurrent Engineering System—The Feature-Oriented Capability Module,'' J. Mater. Process. Technol., **61**, pp. 124–129.
[29] Ingle, T. H., 1998, ''*Leveraging the Learning Process in Manufacturing*,'' MSc, MIT, Cambridge, MA.
[30] DiCarlo, L., 1998, ''Manufacturing is Key to Chip Success,'' *PC Week*, May 11, 1998, **15** No. 19, pp. 33.
[31] Haber, C., 1998, ''AMD's Last Chance,'' *Electronic News*, April 13, 1998 **44**, No. 2214, 1.
[32] Fine, C. H., 1986, ''Quality Improvement and Learning in Productive Systems,'' Manage. Sci., **32**, No. 10. pp. 1301–1315.
[33] Clark, K. B., and Fujimoto T. 1991 *Product Development Performance: Strategy, Organization, and Management in the World Auto Industry*, Harvard Business School Press, Boston.
[34] Womack, J. P., Jones, D. T., and Roos, D., 1991, *The Machine That Changed the World: The Story of Lean Production*, Harperperennial Library, New York.
[35] Rothschild, M., and Stiglitz, J. E., 1970, ''Increasing Risk: I. A Definition,'' J. Economic Theory, **2**, pp. 225–243.
[36] Shannon, C. E., and Weaver, W., 1949, ''*The Mathematical Theory of Communication*,'' The University of Illinois, Urbana, Illinois.

[37] Kelly, J. L. J., 1956, ''A New Interpretation of Information Rate,'' Bell Syst. Tech. J., **35**, pp. 917–926.
[38] Ertan, B., 1998, ''*Analysis of Key Characteristic Methods and Enablers Used in Variation Risk Management*,'' MSc, MIT, Cambridge.
[39] Banks, H., 1998, ''Slow Learner,''*Forbes*, May 4, 1998, p. 54.
[40] Craig, M., 1996, ''Dimensional Management, Not Tolerance Assignment,'' Manuf. Eng., **117**, No. 6, p. 8.
[41] Jacobs, A., 1998, ''Users Warming to sub-$1000 PCs,'' *Computerworld*, August 3, p. 53.
[42] Weil, N., 1998, ''Sub-$1000 PC Market Set to Boom,''*IDG News Service* May 13, 1998.
[43] Otto, K., 1994, ''Robust Product Design Using Manufacturing Adjustments,'' *Design Theory and Methodology*, ASME Design Engineering Technical Conferences, Minneapolis, MN, pp. 1–8.