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A Mathematical Framework for the Key Characteristic Process

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Abstract. *To maximize product quality, a product design team selects concepts and dimensions to minimize a product's sensitivity to variation. However, even for the most robust products, it is rarely possible to transition a product into production without encountering any variation-related problems. In a complex product, it is not economically or logistically feasible to control and/or monitor the thousands of tolerances specified in a product's drawing set. To address this problem, many organizations are using Key Characteristic (KCs) methods to identify where excess variation will most significantly affect product quality, and what product features and tolerances require special attention from manufacturing. As simple as this principle seems, most companies struggle to effectively implement KC methods because no quantitative methods to prioritize KCs exist. This paper develops a mathematical definition of a KC based on a variation propagation model. In addition, it develops a quantitative effectiveness measure used to prioritize where verification, variation reduction, and on-going monitoring should be applied. The effectiveness measure incorporates the cost of control, the benefit of control, and the expected change in process capability. The methods are illustrated using an automotive door assembly.*

Keywords: Key characteristics, Process verification, Quality monitoring; Variation modeling

1. Introduction

To maximize product quality, a product design team selects concepts and dimensions to minimize a product's sensitivity to variation. However, even for the most robust products, it is rarely possible to transition a product into production without encountering any variation-related problems. It is still necessary during transition to production to verify that processes conform to requirements and to remove excess variation. During production, it is necessary to continually monitor processes to ensure that processes don't degrade. However, it is not economically or logistically feasible to control and/or monitor all of the thousands of tolerances specified in a drawing set.

To address the problem of what tolerances to verify, improve, and control, many organizations are using special designations called Key Characteristics (KC), critical parameters and special characteristics. Key Characteristics drawing designations are used to indicate where excess variation will affect product quality and what product features and tolerances require special attention from manufacturing (Lee and Thornton 1996).

As simple as this principle seems, most companies struggle to effectively implement KC methods. We have heard the same questions being asked at many companies. *What does 'critical' mean? What are the criteria for designating a KC? What does it mean when something is designated a KC? Is measurement required? How much variation do we need to remove?*

Despite the ongoing debate, no clear answer has emerged. We believe the continued confusion about KCs is rooted in the debate format, which has focused on semantics and terminology, rather than on quantitative engineering analysis. This paper answers these questions using a quantitative engineering-based framework. The quantitative model is based on four assumptions. First, quality is evaluated after a product or subsystem is assembled but variation is controlled when the part is manufactured and/or when the part is installed. Secondly, resources for variation control are limited. Thirdly, limited resources necessitate ranking the individual part and process characteristics. Fourthly, the ranking is a function of how sensitive a product is to a change in process capability, the expected change in process capability, and the variation control costs.

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This paper first reviews the current KC methodologies and their weaknesses (Section 2). Secondly, it develops a quantitative effectiveness measure used to prioritize where verification, variation reduction and ongoing monitoring should be implemented. The effectiveness measure is based on the physics of the product, the cost of control, the benefit of control, and the expected change in process capability (Section 4).

2. Current Practice

Variation in parts and processes increase cost and decreases product quality (Deming 1986; Taguchi and Clausing 1990). However, it is impossible to reduce variation to zero – the cost would be prohibitive. Consequently, the design team must balance variation cost against the cost to increase quality (termed 'tolerance design' by Taguchi (1993)). The result of the cost/quality trade-off is the allowable variation (also termed latitude). Latitude is expressed through tolerances placed on specifications and drawings. Manufacturing is subsequently responsible for ensuring conformance to these tolerances.

Because manufacturing can not monitor and track all tolerances, companies use KC designations to focus manufacturing attention on the critical few. Most large design and manufacturing firms have a KC methodology. KC processes are outlined in company specific guides, which describe the steps to create and react to a KC designation. All guides have similar content and format. The following section describes a generic KC methodology including definitions, KC flowdowns, and methods to react to KC designations.

2.1. Example Definitions

Through interviews and work with a variety of companies, a large list of definitions was collected. The following are typical examples.

> *Key Characteristics are designated to identify those part or assembly features/interfaces where variation from nominal results in the greatest loss, Statistical Process Control (SPC) measurements are focused on these characteristics to minimize variation, ensure capable processes, and reduce unnecessary inspection requirements.*
>
> *A feature becomes a Key Characteristic if the variation from its nominal value has significant effect on fit, performance, or service life of the product.*
>
> *Key Characteristics are the measurable design variables that directly affect the performance of a system/subsystem.*
>
> *Key Characteristics are those features that are important to customer satisfaction and require special control.*
>
> *Identifying features as important or critical does not make them good Key Characteristics. If manufacturing can not economically measure and chart such features, then the basic requirement of a Key Characteristic – statistical control and process capability can not be demonstrated.*

The definitions have several common themes. All definitions relate KCs to variation from nominal and specify that variation must create a loss. Some definitions state that a KC designation implies additional resources are needed to control variation. In addition, according to some definitions, features should be ranked: those features whose variation impact is greatest should be designated as KCs. We propose a hybrid definition:

> *Key Characteristics are the product, sub-assembly, part, and process features that significantly impact the final cost, performance, or safety of a product when the KCs vary from nominal. Special control should be applied to those KCs where the cost of variation justifies the cost of control.*

2.2. KC Flowdown

Most KC methods are based on the concept of a KC flowdown. The KC flowdown provides a system view of potential variation risk factors and captures the design team's collective knowledge about variation and its contributors. A KC flowdown is the hierarchy of variation-sensitive product requirements and part and process features that contribute to their variation. The term KC has been applied to both variation sensitive product requirements and individual part dimensions that introduce variation. Figure 1 shows a KC flowdown for a car door.

One key customer requirement is the perception of the car door. Several physical characteristics (product-KCs) of the car door influence the customer's perception of quality. Among these are the evenness of the seams, steps between the panels, and the door closing force. Each product-KC has several contributing subsystem-KCs (e.g. outer perimeter of the door, body aperture, etc.). These, in turn, are a function of the part-KCs (the door panel shape) and process-KCs (the fixtures and stamping processes).

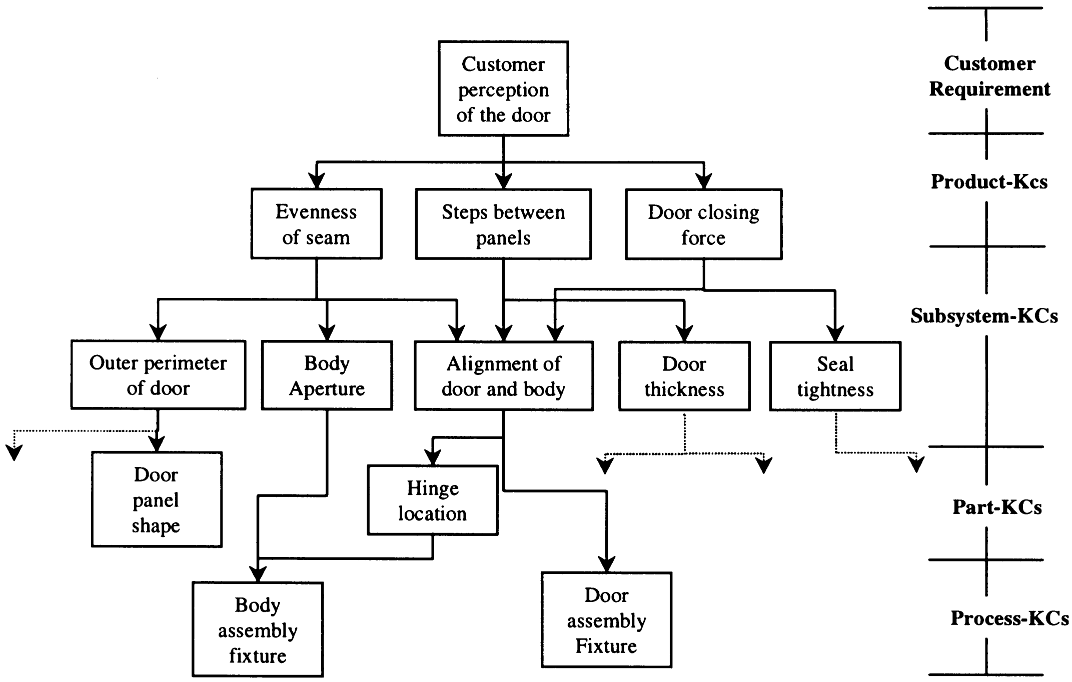

Figure 1. KC flowdown.

Traceability between part- and product-KCs through subsystem-KCs is necessary to identify what features should be controlled, identify where changes in processes may impact the product quality, and identify root causes of quality problems.

2.3. KC Process

Figure 2 shows a diagram of a typical KC process. This process is a generalization of many company processes. Most KC processes can be broken down into two sections. During design, the design team *identifies* the KCs, sets the allowable latitudes, and creates the KC flowdown (top three boxes). The KC flowdown is based on engineering requirements, engineering knowledge, product architecture, and assembly process and is typically developed by the entire design team using a QFD(Quality Function Deployment)-like process.

After the potential contributors to variation are identified, the product development team must decide how to *react* to the KCs. These decisions are made when the product is transitioned from design to production. The team must identify the sub-set of KCs requiring verification, reduction, and/or monitoring. Two data sets are used to select the reaction plan: process capability data and the variation model.

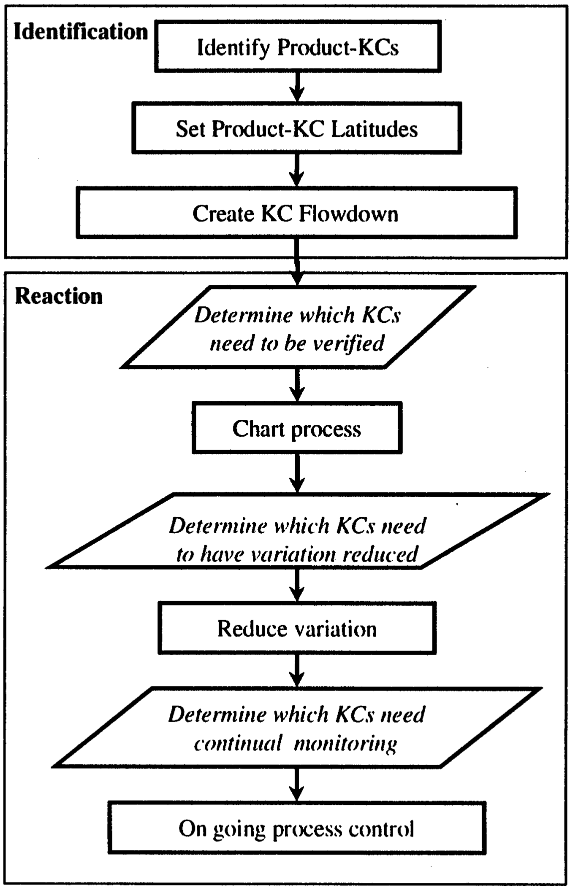

Figure 2. Typical KC processes.

2.3.1. Identification

The first step is the identification of the product-KCs (the variation-sensitive product requirements). In addition, design teams should quantify the acceptable latitude and cost incurred when variation exceeds allowable variation. Manufacturing uses the latitudes as benchmarks against which product quality is measured. Once product-KCs have been identified, sub-system-, part- and process-KCs are identified and captured in a KC flowdown. A variety of tools have been used to capture KC flowdowns including drawings, databases and spreadsheets.

2.3.2. Reaction

Most variation management methods describe the use of three methods: verify variation, reduce variation, or monitor variation.

- **Verification.** Tolerances are specified such that, if they are achieved, the product will have acceptable quality. However, when ramping up a new production process, it is uncertain whether production equipment will achieve the tolerances without tuning or adjustment. If the standard deviation or bias is too high and process capability is not measured, it may be hard to find root causes of quality problems. To verify product quality, early production runs may be subjected to stringent measurement processes. However, even if all of part-KC processes are on target, product-KCs may still exhibit unacceptable levels of variation. This may occur when either the model is incorrect, or there are unexpected sources of variation. In either case, tolerances and/or processes may require adjustment.
- **Variation reduction.** Once process capability has been assessed, some part and process-KCs may require variation reduction. Variation Reduction (VR) applies to a broad set of tasks performed by process improvement teams. After the VR teams target a part, process, or sub-system they perform root cause analyses to understand major contributors to the variation problem. Process adjustment, machine upgrades, machine maintenance, standard processes implementation, and/or employee training are then used to reduce variation.
- **On-going monitoring.** Process degradation is a fact in production: tools wear, operators take shortcuts, and suppliers change. Statistical Process Control (SPC) and inspection are commonly used to detect degradation before it impacts final product quality. Because SPC and inspection can be resource intensive, manufacturing cannot monitor all tolerances. Therefore, SPC and inspection are typically placed where both the cost and probability of degradation are high.

2.3.3. Process capability databases and variation models

Design teams use two sets of information to select KCs: variation models and process capability data.

- **Variation models.** Manufacturing processes inevitably introduce variation into individual part dimensions. As a product is assembled, individual errors combine and the total effect of errors is seen when assembly is complete. It is necessary to have a model of variation to predict final product quality. Several papers discuss using product variation models to calculate optimal tolerances (Dong and Shi 1997; Sudarsan et al. 1998), predict yields (Kazmer et al. 1996; Zemel and Otto 1996), or model the effects of process capability uncertainty (Thornton 1999). Where a quantitative model is not available or deemed too difficult to develop, Design of Experiments (DOE) provides a systematic method to identify the relative contribution of input parameters using a physical prototype (Phadke 1989).
- **Process capability data.** The planning for variation control also requires process capability knowledge. Most companies use one of two methods to predict process capability: process capability databases and manufacturing knowledge. Process capability databases store data collected while monitoring production of similar products (Lucca et al. 1995; Naish 1996; Tata and Thornton 1999). Manufacturing knowledge is the process information informally maintained by the manufacturing, process and production engineers.

2.4. Summary

In many companies, a KC designation on a drawing automatically requires implementation of verification, reduction, and on-going monitoring. Consequently, production is often overwhelmed with KCs and will use their own judgement to select where control is imposed. Several problems contribute to the inability to prioritize KCs: no systematic flowdown, no product variation model, and no standard quantification model.

In many cases, companies do not systematically flow KCs through the product. Rather, they start by identifying product requirements then they identify, based on their engineering judgement, what individual features contribute to product requirements without tracing through the subsystem-KCs. When a design team fails to create a systematic flowdown, too many KCs are identified, it can be hard to trace root causes (Lee and Thornton 1996), and quality plans can become too cumbersome.

Secondly, most companies do not have common repositories for product models. Because the relationship between product-KCs and part-KCs are not easily found, companies often struggle with tracing variation sources (Ertan 1998; Leland 1997) and have no quantitative method for prioritizing KCs.

Finally, most design teams use two heuristics to select KCs. First, most KC guidelines tell designers to designate a KC where there is a steep loss function. This approach can be used to prioritize the product requirements but can't be applied to the part and process features. Secondly, the part-KCs are often prioritized based on their relative contribution. However, this is not consistent with the requirements for verification and monitoring, which are related to reducing variation and/or the uncertainty about variation.

3. Quantitative KC Model

As stated above, KC methods are used to identify the part-KCs where control – verifying, improving, and monitoring – is most effective. The effectiveness is a function of the cost and benefit of control. The model presented in this paper prioritizes where control is most effective. The paper derives an effectiveness measure, E_{ij} – a non-dimensional number that quantifies the improvement in the product-KC i enabled by controlling part-KC j. The effectiveness measure is analogous to the FMEA Risk Priority Number (RPN). RPN is a function of a potential failure mode's severity, occurrence and detectablity. The effectiveness measure is a function of the variation sensitivity, magnitude of variation, and cost of controlling variation. Both RPN and E are used to rank order where effort will best reduce the risk of failure. However, E, unlike RPN, is derived from engineering principles.

The following outlines the overall derivation detailed in Sections 3.1–3.4. First, we model a quantitative measure of *benefit*. The most commonly used quality cost metric is the Taguchi loss function. Although the Taguchi function tends to overestimate quality costs, it provides a good measure of relative quality. The loss, L_i, for product-KC i is a function of its standard deviation, σ_i, its bias, b_i, cost constant, k_i, the cost of failure, C_i, and the upper and lower specification limits, UL_i and LL_i

$$L_i = k_i(b_i^2 + \sigma_i^2) \text{ where } k_i = \frac{C_i}{\left(\frac{UL_i - LL_i}{2}\right)^2} \qquad (1)$$

The costs of failure, C_i, and the upper and lower limits UL_i and LL_i are set based on historical knowledge and understanding of customer needs. The higher the value of C_i, the more important the product-KC. For example, a safety related product-KC would typically have a higher cost of failure than an aesthetic-related product-KC. In most cases, it is difficult to measure the exact cost of quality. Because the costs are used to create a ranking, it is only necessary to ensure that the relative values of cost are consistent.

The purpose of this paper is to identify which part-KCs impact the overall product quality the most. To do this we must first derive the *loss-sensitivities:*

$$\frac{\partial L_i}{\partial \sigma_{j\ell}} \text{ and } \frac{\partial L_i}{\partial b_{j\ell}} \qquad (2)$$

This measure quantifies the controllability of the product for each part-KC. The larger the value of $\partial L_i/\partial\sigma_{j\ell}$, the more benefit will accrue from controlling part-KC i. The *benefit* of controlling part-KC i, ΔL_{ij}, is a function of the sensitivities, the change in standard deviation from $\sigma_{j\ell}$ to $\sigma_{j\ell}$, and the change in bias from $b_{j\ell}$ to $b_{j\ell}$. ΔL_{ij} is calculated by integrating Eq. (3)[2]:

$$\Delta L_{ij} = \int_{\sigma_{j\ell}}^{\sigma'_{j\ell}} \frac{\partial L_i}{\partial \sigma_{j\ell}} d\sigma_{j\ell} + \int_{b_{j\ell}}^{b'_{j\ell}} \frac{\partial L_i}{\partial b_{j\ell}} d\mu_{j\ell} \qquad (3)$$

For each of the three stages of KC reaction – verification, reduction, and ongoing monitoring – the values of $\sigma_{j\ell}$, $\sigma_{j\ell}$ and $b_{j\ell}$ to $b_{j\ell}$ are different. In verification, $\Delta\sigma_{j\ell}$ is the difference between desired standard deviation and the standard deviation produced by manufacturing. In variation reduction, it is the expected improvement in process variation. In monitoring, it is the change in process capability due to degradation.

Finally, to measure effectiveness, the benefit must be balanced against the *cost* to control variation, I_j. I_j is the per-part cost of variation control (including both

[2]The linerization simplified the calculation and did not change the final analysis. The relative values of E_{ij} are used to prioritize the part-KCs. Ignoring the non-linear terms did not impact the rank ordering.

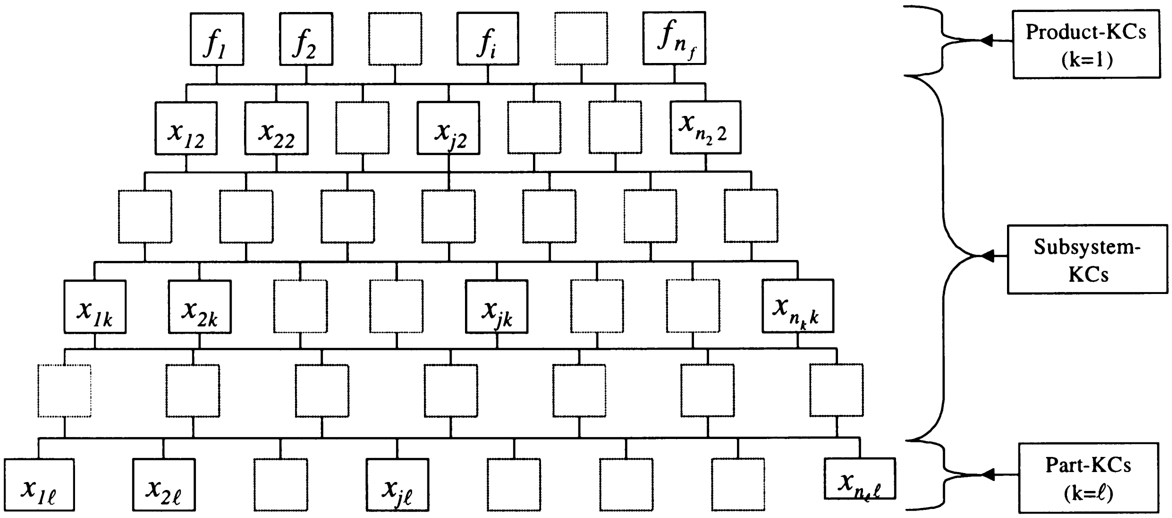

Fig. 3. Generalized KC flowdown.

recurring and fixed costs). The effectiveness measurement is essentially a measure of return on investment:

$$E_{ij} = \frac{\Delta L_{ij}}{I_j} \tag{4}$$

This measure only requires the investment and loss values to be self-consistent. Because ΔL_{ij} and I_j may not be consistent with each other, a value of $E_{ij} > 1$ does not guarantee a positive return on investment. Rather, E_{ij} is used to rank order the part-KCs. The next sections detail the derivations of Eqs (1)–(4).

3.1. Variation Model

This section derives the quantitative relationship between product-KC variation (σ_i and b_i) and part-KC variation ($\sigma_{j\ell}$ and $b_{j\ell}$). This section derives two constants, D_{ij} and T_{ij}: the product-KC, i, to part-KC, j variation sensitivities. The result of this derivation is:

$$b_i = \sum D_{ij} b_{j\ell} \text{ and } \sigma_i^2 = \sum T_{ij} \sigma_{j\ell}^2 \tag{5}$$

Figure 3 is a generalized version of the KC flowdown in Fig. 1. The top layer of the flowdown has n_f product-KCs. Each has a nominal value, f_i, a tolerance, $[LL_i, UL_i,]$, a standard deviation, σ_i, a mean, b_i, and a failure cost, C_i. Product-KCs are created by ℓ layers of subsystem-, part- and process-KCs, x_{jk} (the jth parameter in the kth level). Each layer, k, has n_k KCs.

Using Taylor expansions, the expression relating deviations in a product-KC, Δf_i, to the next KC layer, $\Delta \mathbf{x_2}$ is represented as a linear equation[3]. The linearization assumption does not significantly impact the predicted values. Δx_{ij} is typically very small, and the value of $(\Delta x_{ij})^2$ can be considered insignificant. In addition, the functions can be considered monotonic around the nominal value:

$$\Delta f_i = \frac{\partial f_i}{\partial x_{12}} \Delta x_{12} + \frac{\partial f_1}{\partial x_{22}} \Delta x_{22} + \ldots + \frac{\partial f_1}{\partial x_{n_2 2}} \Delta x_{n_2 2} \tag{6}$$

$\delta f_i / \delta x_{j2}$ quantifies the sensitivity of a product-KC, f_i, to a change in subsystem-KC, Δx_{j2} (termed variation-sensitivity). The array subsystem-KC deviations, $\Delta \mathbf{x_2}$, can be related to the product-KCs array, $\Delta \mathbf{f}$, through the matrix of partial derivatives, $\boldsymbol{\delta_1}$. A matrix approach to variation propagation and modeling has been used extensively by authors such as Homann and Thornton (1998), Slocum (1992), Suh (1990), Frey et al. (1998) and Gao et al. (1998).

$$\Delta \mathbf{f} = \boldsymbol{\delta_1} \Delta \mathbf{x_2} \text{ where } \boldsymbol{\delta_1} = \begin{bmatrix} \frac{\partial f_1}{\partial x_{12}} & \frac{\partial f_1}{\partial x_{22}} & \cdots & \frac{\partial f_1}{\partial x_{n_2 2}} \\ \frac{\partial f_2}{\partial x_{12}} & \frac{\partial f_2}{\partial x_{22}} & \cdots & \frac{\partial f_2}{\partial x_{n_2 2}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_{n_1}}{\partial x_{12}} & \frac{\partial f_{n_1}}{\partial x_{22}} & \cdots & \frac{\partial f_{n_1}}{\partial x_{n_2 2}} \end{bmatrix} \tag{7}$$

[3]Based on discussions and white paper by Srinivasan (1998).

For each subsequent layer, Eq. (6) can be generalized to:

$$\Delta x_{jk} = \frac{\partial x_{jk}}{\partial x_{1(k+1)}}\Delta x_{1(k+1)} + \frac{\partial x_{jk}}{\partial x_{2(k+1)}}\Delta x_{2(k+1)} + \ldots + \frac{\partial x_{jk}}{\partial x_{n_{(k+1)}(k+1)}}\Delta x_{n_{(k+1)}(k+1)} \quad (8)$$

The matrix, $\boldsymbol{\delta}\mathbf{k}$, relates $\Delta \mathbf{x_{k+1}}$ to $\Delta \mathbf{x_k}$

$$\Delta \mathbf{x_K} = \boldsymbol{\delta}_\mathbf{K} \Delta \mathbf{x_{(K+1)}}, \text{ where } \boldsymbol{\delta}_\mathbf{K} = \begin{bmatrix} \frac{\partial x_{1k}}{\partial x_{1(k+1)}} & \frac{\partial x_{1k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{1k}}{\partial x_{n_{k+1}(k+1)}} \\ \frac{\partial x_{2k}}{\partial x_{1(k+1)}} & \frac{\partial x_{2k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{2k}}{\partial x_{n_{k+1}(k+1)}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial x_{n_k k}}{\partial x_{1(k+1)}} & \frac{\partial x_{n_k k}}{\partial x_{2(k+1)}} & \cdots & \frac{\partial x_{n_k k}}{\partial x_{n_{k+1}(k+1)}} \end{bmatrix} \quad (9)$$

The correlation between variation in process- and part-KCs[4] (represented by array $\mathbf{x}_\ell$) and product-KCs can be expressed by:

$$\Delta \mathbf{f} = \boldsymbol{\delta}_1 \boldsymbol{\delta}_2 \boldsymbol{\delta}_3 \ldots \boldsymbol{\delta}_{\ell-1} \Delta \mathbf{x}_\ell \quad (10)$$

To simplify the representations, matrix **D** is used.

$$\Delta \mathbf{f} = \mathbf{D} \Delta x_\ell, \textit{where } \mathbf{D} = \boldsymbol{\delta}_1 \boldsymbol{\delta}_2 \boldsymbol{\delta}_3 \ldots \boldsymbol{\delta}_{(\ell-1)} \quad (11)$$

The analysis in Eqs (6)–(11) models the errors in a single product. In addition, the statistical behavior can be modeled using the same equations. The standard deviation, σ_{jk} and bias b_{jk}, for any KC, x_{jk} is:

$$\sigma_{jk}^2 = \left(\frac{\partial x_{jk}}{\partial x_{1(k+1)}}\right)^2 \sigma_{1(k+1)}^2 + \left(\frac{\partial x_{jk}}{\partial x_{2(k+1)}}\right)^2 \sigma_{2(k+1)}^2 + \ldots + \left(\frac{\partial x_{jk}}{\partial x_{n_{(k+1)}(k+1)}}\right)^2 \sigma_{n_{(k+1)}(k+1)}^2 \quad (12)$$

$$b_{jk} = \frac{\partial x_{jk}}{\partial x_{1(k+1)}} b_{1(k+1)} + \frac{\partial x_{jk}}{\partial x_{2(k+1)}} b_{2(k+1)} + \ldots + \frac{\partial x_{jk}}{\partial x_{n_{(k+1)}(k+1)}} b_{n_{(k+1)}(k+1)} \quad (13)$$

Equations (12) and (13) can also be represented in a matrix form, but a second matrix, $\tau_\mathbf{k}$, is needed to calculate standard deviation.

$$\boldsymbol{\tau}_\mathbf{K} = \begin{bmatrix} \left(\frac{\partial x_{1k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{1k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{1k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \\ \left(\frac{\partial x_{2k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{2k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{2k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial x_{n_k k}}{\partial x_{1(k+1)}}\right)^2 & \left(\frac{\partial x_{n_k k}}{\partial x_{2(k+1)}}\right)^2 & \cdots & \left(\frac{\partial x_{n_k k}}{\partial x_{n_{k+1}(k+1)}}\right)^2 \end{bmatrix} \quad (14)$$

[4]For simplicity, the lowest level on the KC flowdown will be referred to as part-KCs. However, they may include dimensions, tools or processes.

T is the result of ℓ-1$\tau_\mathbf{i}$ matrices being multiplied:

$$\mathbf{T} = \boldsymbol{\tau}_1 \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 \ldots \boldsymbol{\tau}_{(\ell-1)} \quad (15)$$

When Eqs (12), (13) and (15) are combined, the product variation can be predicted:

$$\mathbf{b_f} = \mathbf{D}\mathbf{b}_\ell \textit{ and } \boldsymbol{\sigma}_\mathbf{f}^2 = \mathbf{T}\boldsymbol{\sigma}_\ell^2 \quad (16)$$

$\sigma_\mathbf{f}$ and $\mathbf{b_f}$ are the product-KC arrays and σ_ℓ and $\mathbf{b}_\ell$ are the part-KCs arrays. In most cases $D^2_{ij} = T_{ij}$, except when a part- or process-KC contributes multiple times to the same product-KC. This may happen, for example, if a single worn tool is used to make multiple parts for an assembly and, consequently, all parts in the assembly are undersized.

To use this matrix, it is necessary to calculate the individual sensitivities. While the numbers of sensitivities may initially appear large, the sensitivities are relatively straightforward to estimate. First, the matrices are relatively sparse. If there is no relationship between feature *i* and feature *j*, the sensitivity is zero. Secondly, for geometric assemblies, most sensitivities will have a value of 1 except where an angle is involved. This method can also be used with a black box model such as Variation Systems Analysis. In this case, the sensitivities can be estimated by making small changes in the dimensions of the part recording their effects.

Referring back to Fig. 2, there were two data sets used throughout the KC process: process capability databases and product models. These two databases correspond to values in Eq. (16) **T** and **D** matrices are derived from the variation model and the σ_ℓ and $\mathbf{b}_\ell$ arrays are derived from process capability databases.

3.2. Loss Function

The loss function, L_i, defined in Eq. (1) is combined with Eq. (5) to calculate the product loss from the variation model and part-KC process capability:

$$L_i = k_i \left[\left(\sum_{j=1}^{n_\ell} D_{ij} b_{\ell j} \right)^2 + \sum_{j=1}^{n_\ell} T_{ij} \sigma_{\ell j}^2 \right] \quad (17)$$

In most cases, a product will have more than one product-KC ($n_f > 1$) each of which has a unique k_i value. In addition, part-KCs may contribute to more than one product-KC (T_{ij}>0 for more than one *j*). The total product loss, L_T, is the sum of the individual loss functions, L_i:

$$L_T = \sum_{i=0}^{n_f} k_i \left[\left(\sum_{j=1}^{n_\ell} D_{ij} b_{j\ell} \right)^2 + \sum_{j=1}^{n_\ell} T_{ij} \sigma_{j\ell}^2 \right] \quad (18)$$

3.3. Loss-sensitivity

The loss-sensitivity for a single product-KC is calculated by taking the partial derivative of loss with respect to $\sigma_{j\ell}$ and $b_{j\ell}$.

$$\frac{\partial L_i}{\partial \sigma_{j\ell}} = 2k_i T_{ij} \sigma_{j\ell} \text{ and } \frac{\partial L_i}{\partial b_{j\ell}} = 2k_i D_{ij} b_i \quad (19)$$

If a part-KC contributes to more than one product-KC, the total loss-sensitivity is:

$$\begin{aligned} \frac{\partial L_T}{\partial b_{j\ell}} &= 2\left(b_1 k_1 D_{1j} + b_2 k_2 D_{2j} + \ldots + b_{n_f} k_{n_f} D_{n_f j} \right) \\ \frac{\partial L_T}{\partial \sigma_{j\ell}} &= 2\sigma_{j\ell}\left(k_1 T_{1j} + k_2 T_{2j} + \ldots + k_{n_f} T_{n_f j} \right) \end{aligned} \quad (20)$$

3.4. Effectiveness Measure

As stated above, there are three steps in variation control: verification, reduction and monitoring. This section derives the effectiveness measures E_j and E_{ij} for each process: E_{ij} is the impact of controlling part-KC j on product-KC i and E_j is the effectiveness of controlling part-KC j on the total product quality.

3.4.1. Verification

KC verification is used in the first stages of production to ensure a production process meets the required targets. This information is used in later steps to prioritize where variation reduction efforts are allocated.

The design is defined assuming a standard deviation of $\sigma_{j\ell}$ and bias of $b_{j\ell}$ can be achieved. However, in many cases, processes are likely to introduce more variation than expected. The worst case variation is defined by $\sigma_{j\ell(uc)}$ and $b_{j\ell(uc)}$. The upper limit and lower values of the standard deviation and mean shifts can often be found in process capability databases (Tata and Thornton 1999), or through focused process knowledge. Total cost due to unforeseen variation, $\Delta L_{j(uc)}$, is calculated by:

$$\Delta L_{j(uc)} = \int_{\sigma_{j\ell}}^{\sigma_{j\ell(uc)}} \frac{\partial L_T}{\partial \sigma_{j\ell}} d\sigma_{j\ell} + \int_{b_{j\ell}}^{b_{j\ell(uc)}} \frac{\partial L_T}{\partial b_{j\ell}} db_{j\ell} \quad (21)$$

Assuming T_{ij} is constant and does not vary with a change in standard deviation:

$$\Delta L_{j(uc)} = \sum_{i=0}^{n_f} k_i \left(T_{ij} \left(\sigma_{j\ell(uc)}^2 - \sigma_{j\ell}^2 \right) + D_{ij}^2 \left(b_{j\ell(uc)}^2 - b_{j\ell}^2 \right) \right) \quad (22)$$

To calculate verification effectiveness, the loss in Eq. (22) is divided by the verification cost, $I_{j(ver)}$.

$$E_{ij(ver)} = \frac{k_i}{I_{j(ver)}} \left(T_{ij} (\sigma_{j(uc)}^2 - \sigma_j^2) + D_{ij}^2 (b_{j(uc)}^2 - b_j^2) \right) \quad (23)$$

The total effectiveness is:

$$E_{j(ver)} = \sum_{i=1}^{n_f} \frac{k_i}{I_{j(ver)}} \left(T_{ij} (\sigma_{j(uc)}^2 - \sigma_j^2) + D_{ij}^2 (b_{j(uc)}^2 - b_j^2) \right) \quad (24)$$

3.4.2. Reduction

As stated above, variation reduction is the systematic process of diagnosing and removing sources of excess variation. The following analysis assumes that the cost of reducing the bias is small compared to reducing standard deviation. Therefore, biases are not included in this analysis.

One of two methods can be used to derive the effectiveness measure for variation reduction efforts: continuous and discrete. The first assumes a team can choose the desired improvement level and the variation reduction cost increases exponentially as the desired standard deviation decreases. The second assumes the available variation reduction, $\Delta\sigma_j$, and the cost $I_{j(vr)}$ are constant. Both were found to have similar resultant equations. For simplicity, the discrete case is used.

In the discrete case, it is assumed that the variation reduction $(\sigma_j - \sigma_{j(vr)})$ and costs $I_{j(vr)}$ are fixed. The effectiveness of a discrete change in variation is therefore

$$E_{ij(vr)} = \frac{k_i}{I_{j(vr)}} T_{ij} (\sigma_j^2 - \sigma_{j(vr)}^2) \quad (25)$$

The total effectiveness measure is:

$$E_{j(vr)} = \frac{1}{I_{j(vr)}} \sum_{i=0}^{n_f} k_i T_{ij} (\sigma_j^2 - \sigma_{j(vr)}^2) \quad (26)$$

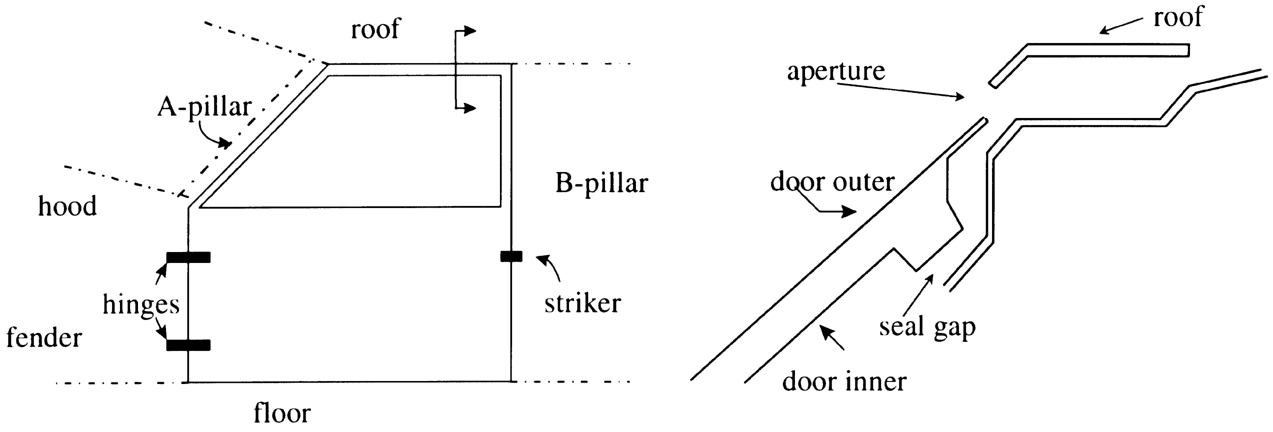

Fig. 4. Generic automotive door and upper.

3.4.3. Monitoring
Once processes have been qualified and variation reduction efforts completed, some processes may require ongoing monitoring and control to ensure that the quality of the product does not degrade. Inspection or SPC are required where the expected process degradation will significantly impact a product's final quality; for example, control may be required where tools wear, settings drift, or input materials change.

First, the team must determine the potential degradation, $\sigma_{j\ell(d)}$ and $b_{j\ell(d)}$. Monitored processes are prioritized based on the loss incurred by degradation. Using the same derivation:

$$E_{ij(m)} = \frac{k_i}{I_{j(m)}}\left(T_{ij}(\sigma^2_{j(d)} - \sigma^2_j) + D^2_{ij}(b^2_{j(d)} - b^2_j)\right) \quad (27)$$

The total effectiveness measure for monitoring can be expressed by:

$$E_{j(m)} = \sum_{i=1}^{n_f} \frac{k_i}{I_{j(m)}}\left(T_{ij}(\sigma^2_{j(d)} - \sigma^2_j) + D^2_{ij}(b^2_{j(d)} - b^2_j)\right) \quad (28)$$

3.4.4. Summary
All three effectiveness measures have similar structures. The effectiveness of control increases with the cost of failure, the sensitivity of the product to part-KC j, the magnitude of the process variation, and the magnitude of the change in the process variation enabled by the control strategy. The effectiveness decreases as the cost to control increases.

4. Car Door Case Study

A car door, based on Leland's (1997) case study[5], will be used to illustrate the application of the effectiveness measures. Geometry and numbers have been significantly modified to mask any proprietary data. First, the example will be introduced. Secondly, the **D** and **T** matrices will be derived. Thirdly, the relative rankings will be generated. Finally, a method to select the KCs to verify, improve, and monitor is proposed.

4.1. Introduction

Car doors are complex sub-systems that are critical to the car's total quality. Together with other subsystems such as the fender, roof, and B-pillar-doors-create many KCs important to body quality. Figure 4 shows a generic door and a detail of the door/roof interaction. The door/roof interaction is a critical area as it contains at least three separate KCs: seal gap, steps and gaps. Seal gap is the distance between the door and body when the door is closed. If the seal gap is too small, door-closing force will be too high. If seal gap is too large, wind noise in the car interior becomes a problem. Secondly, the gap between the roof and door outer (termed the aperture) is important to the customer, as excess or uneven gaps are viewed as an indicator of low quality. Thirdly, the flushness (steps) between the roof and door outer and roof is also critical to quality. In Leland (1997), the case study subject spent significant time and resources tracing and fixing problems associated with excess variation in all three KCs.

[5]Supervised by the author.

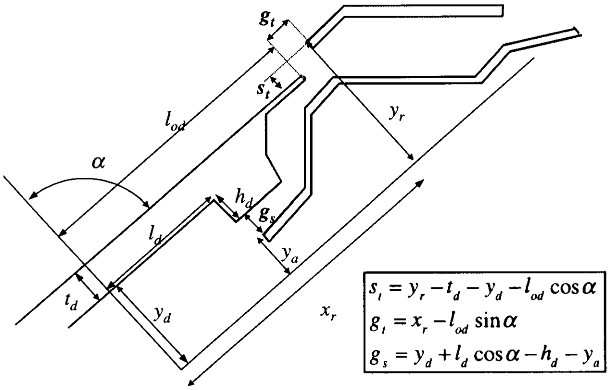

Fig. 5. Upper door detail.

4.2. Variation Model

Figure 5 shows a dimensioned view of the upper door. Three KCs – seal gap, g_s, step, s_t, and gap, g_t – between the door and roof are indicated in bold. All three are affected by a combination of the roof location (x_r and y_r), body location (y_a), door location (y_d and α) and door geometry (l_{od}, l_d, t_d, and h_d). The figure also shows the quantitative relationship between part-KCs to product-KCs.

4.3. D and T Matrices

The **f** and $\mathbf{x_2}$ arrays are:

$$\mathbf{f} = [g_t \quad s_t \quad g_s]^T \text{ and } \mathbf{x}_2 = [l_d \quad l_{od} \quad h_d \quad t_d \quad y_d \quad y_a \quad y_r \quad x_r \quad \alpha]^T \tag{29}$$

The **D** matrix is derived by applying Eq. (7) to the equations in Fig. 5.

$$\mathbf{D} = \begin{bmatrix} 0 & -\sin\alpha & 0 & 0 & 0 & 0 & 0 & 1 & l_{od}\cos\alpha \\ 0 & -\cos\alpha & 0 & -1 & -1 & 0 & 1 & 0 & l_{od}\sin\alpha \\ \cos\alpha & 0 & -1 & 0 & 1 & -1 & 0 & 0 & -l_d\sin\alpha \end{bmatrix} \tag{30}$$

By squaring the **D** matrix elements, the **T** matrix is generated (this is valid because there is only one layer in the KC flowdown):

$$\mathbf{T} = \begin{bmatrix} 0 & (\sin\alpha)^2 & 0 & 0 & 0 & 0 & 0 & 1 & (l_{od}\cos\alpha)^2 \\ 0 & (\cos\alpha)^2 & 0 & 1 & 1 & 0 & 1 & 0 & (l_{od}\sin\alpha)^2 \\ (\cos\alpha)^2 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & (-l_d\sin\alpha)^2 \end{bmatrix} \tag{31}$$

4.4. Example Values

To populate the example, illustrative sample values will be used (these values do not reflect actual capability). Nominal values used for this example are $l_{od} = 10$, $l_d = 8$, and $\alpha = 90^\circ$. Tables 1 and 2 contain the raw data used for the analysis.

Table 1. Product-KC values

		UL_i	LL_i	C_i
g_t	Aperture gap	0.1	–0.1	100
s_t	Aperture step	0.09	–0.09	300
g_s	Seal gap	0.11	–0.11	500

Location parameters were assumed to have a standard deviation of 0.02; door thickness to have a standard deviation of 0.005; and door lengths to have a standard deviation of 0.01. It is also assumed that every part-KC variation can be reduced by 10%. The cost to reduce fixture variation is higher than the cost to reduce part variations. The processes can degrade by either 5% or 10%, and a maximum mean shift degradation of $1.5\sigma_j$[6] is assumed. A summary of the sample values is given in Table 2.

4.5. Results

Throughout this analysis, the three processes – verification, reduction and monitoring were treated separately. However, in reality they are highly coupled. Those features needing ongoing monitoring first require the processes to be verified and improved. If a part-KC has a high E_i value for any process, it will most likely be subjected to all three. The decision not to reduce or monitor a verified part-KC is made after verification is completed and a better understanding of the actual process variation is developed.

To consider this fact, it is necessary to combine the three effectiveness measures. To identify the set that is verified, two values are calculated. The first is the sum of all three effectiveness measures and the second is the total ranking for the three processes. These rankings are used to assess the relative importance of the part-KCs.

The total effectiveness measure, E_j, and rank order for each process were calculated for each part-KC using Eqs (25), (27) and (29) and the values in Table 2. The high priority KCs were found by rank ordering the sum of all three effectiveness measures.

Examining Table 3, l_d, the door inner length, has the lowest total effectiveness measure. This makes sense because l_d does not contribute significantly to any of the product-KCs. y_d has the highest value. This also makes sense because it contributes to both of the most critical product-KCs and its initial variation is

[6]The assumption of a mean shift of 1.5σ is the typical assumption used in the Six Sigma methodologies.

Table 2. Part-KC values

		Target		Actual			Variation Reduction Capability		Degradation $b_{j(uc)=1.5*}\sigma_j$	
		σ_j	b_j	$\sigma_{j(uc)}$	$b_{j(uc)}$	$I_{j(ver)}$	$\sigma_{j(vr)}$	$I_{j(vr)}$	$\sigma_{j(d)}$	$I_{j(m)}$
l_d	Inner Door Length	0.01	0	0.02	0	10	10%	30	5%	10
l_{od}	Outer Door Length	0.01	0	0.02	0	10	10%	30	5%	10
h_d	Stop Height	0.005	0	0.007	0.001	5	10%	30	10%	5
t_d	Door Thickness	0.005	0	0.007	0.001	5	10%	30	10%	5
y_d	Door Location	0.02	0	0.03	0.005	20	10%	40	10%	20
y_a	Stop Location	0.02	0	0.03	0.005	20	10%	40	10%	20
$y_{i;r}$	Roof Location	0.02	0	0.03	0.005	20	10%	40	10%	20
x_r	Roof Location	0.02	0	0.03	0.005	20	10%	40	10%	20
α	Door Location	1E-04	0	0.0002	0.00005	50	10%	60	10%	50

Table 3. Effectiveness matricies

		Verification		*Variation Reduction*		*Monitoring*		$E_{j(ver)} + E_{j(vr)} + E_{j(\mathrm{m})}$	*Total Rank*
		$E_{j(ver)}$	Rank	$E_{j(vr)}$	Rank	$E_{j(\mathrm{m})}$	Rank		
l_d	Inner Door Length	**0.00**	9	**0.00**	9	**0.00**	9	***0.00***	27
l_{od}	Outer Door Length	**0.30**	4	**0.01**	6	**0.16**	7	***0.47***	17
h_d	Stop Height	**0.21**	6	**0.01**	5	**0.35**	4	***0.57***	15
t_d	Door Thickness	**0.19**	7	**0.01**	7	**0.32**	6	***0.51***	20
y_d	Door Location	**2.06**	1	**0.15**	1	**2.68**	1	***4.89***	3
y_a	Stop Location	**1.08**	2	**0.08**	2	**1.41**	2	***2.58***	6
y_r	Roof Location	**0.97**	3	**0.07**	3	**1.27**	3	***2.31***	9
x_r	Roof Location	**0.26**	5	**0.02**	4	**0.34**	5	***0.62***	14
α	Door Location	**0.00**	8	**0.00**	8	**0.00**	8	***0.01***	24

large. However, not all of the rankings are *a priori* obvious. For example, it is not obvious that α should have a low effectiveness as it contributes to two of the three product-KCs.

The analysis grouped the part-KCs into three groups.

1. **High values:** y_d, y_a and y_r.
2. **Mid values:** x_r, h_d, t_d, and l_{od}.
3. **Low values:** l_d and α.

Because of resource availability, more than just the highly effective part-KCs can be controlled, however limited resources prevent the team from controlling all mid-value part-KCs. To select among the mid-value KCs, two commonly used heuristic constraints are applied:

1. Every product-KC should be monitored by at least one part-KC.
2. Every part should have each of its characteristic dimensions measured.

To optimize the measurement plan, a matrix can be employed to ensure coverage of the two guidelines (Table 4). Product-KCs are listed in the rows and the characteristic dimensions in the columns. If y_d, y_a and y_r are the only KCs selected (shown in bold), product-KC g_t is not tracked, nor are the door lengths or thickness. To satisfy the guidelines, l_{od} and h_d are added (shown underlined). The remainder of the part-KCs are ignored (shown in regular text). This analysis should be repeated after verification to re-evaluate what KCs should be subjected to variation reduction and ongoing monitoring.

5. Conclusions

While a design team can maximize robustness to existing variation, it is still necessary to measure, reduce, and control variation in production. Current methods to identify the critical tolerances are based on heuristics and, consequently, are not guaranteed to

Table 4. Measurement selection

		Door			*Roof*	*Frame*
		Length	*Thickness*	*Location*	*Location*	*Location*
g_t	Aperture Gap	l_{od} (0.47)			x_r (0.62)	
s_t	Aperture Step		t_d (0.51)	$\mathbf{y_d}$ **(4.89)** α(0.01)	$\mathbf{y_r}$ **(2.31)**	
g_s	Seal Gap	l_d (0.0)	h_d (0.57)	$\mathbf{y_d}$ **(4.89)** α(0.01)		$\mathbf{y_a}$ **(2.58)**

most effectively use limited variation control resources. To address this problem, this paper presents a quantitative method to maximize the benefit of quality control. The method leverages knowledge of both the product and the processes.

This research contributes three new concepts. First, a linear model is derived which predicts final product variation from part and process variations. Secondly, the relative importance of a part-KC is dependent on the sensitivity of loss to a change in its process capability. Thirdly, resource allocation is based on an effectiveness measure, which is a function of the loss-sensitivity, the magnitude of the process capability change, and the variation control costs. Future work will include replacing the heuristic selection methods with a quantitative method. Methods to identify what subsystem-KCs need to be verified and methods to account for uncertainties in the T_{ij} and D_{ij} values will also be developed.

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Appendix: Nomenclature

α	Door angle
C_i	Cost of exceeding tolerance
δ_1	Matrix of partial derivates which relates product requirements to second level of the KC flowdown
δ_k	Matrix of partial derivatives which relates the k^{th} level of the KC flowdown to the $(k+1)^{th}$ level
$\Delta\mathbf{f}$	Array of deviations in product-KCs
ΔL_{ij}	Loss in product-KC i caused by part-KC j
$\Delta L_{j(uc)}$	Expected loss due to uncertainty
$\Delta\mathbf{x}_2$	Array of deviations in the second layer in KC flowdown
$\Delta\mathbf{x}_k$	Array of deviations in k^{th} layer of the KC flowdown
$\Delta\mathbf{x}_\ell$	Array of deviations in part- and process-KCs
$\mathbf{D}$	Product of $(\ell-1)$ δ_k matrices
D_{ij}	Element of Matrix **D** relating variation in part-KC j to product-KC i
E_{ij}	Effectiveness of controlling KC j on KC i
$E_{ij(m)}$	Effectiveness of monitoring part-KC j on product-KC i
$E_{ij(ver)}$	Effectiveness of verifying part-KC j on product-KC i
$E_{ij(vr)}$	Effectiveness of reducing variation in part-KC j on product-KC i
$E_{j(m)}$	Total effectiveness of monitoring part-KC j
$E_{j(ver)}$	Total effectiveness of verifying part-KC j
$E_{j(vr)}$	Total effectiveness of reducing verification part-KC j
f_i	Nominal value of product-KC i
g_s	Seal gap
g_t	Aperture gap
h_d	Thickness of upper door
I_{dj}	Cost to reduce variation from $\sigma_{j\ell}$ to σ_{dj}
I_j	Cost of controlling part-KC j
$I_{j(m)}$	Cost of monitoring part-KC j
$I_{j(ver)}$	Cost of verifying part-KC j
$I_{j(vr)}$	Cost of variation reduction
κ_j	Cost constant for variation reduction cost curve
k_i	Taguchi loss function constant for product-KC i
ℓ	Number of layers in the KC flowdown
l_d	Length of door inner
L_i	Loss function for product-KC i
LL_i	Lower limit in product-KC i
l_{od}	Length of door outer
L_T	Total product loss
$b'_{j\ell}$	Maximum value of bias in part-KC j
$\mathbf{b_f}$	Array of product-KC biases
b_i	Bias of product-KC i
b_{jk}	Mean of KC j in the k^{th} layer of the KC flowdown
$b_{j\ell}$	Mean of part-KC j
$b_{j\ell(d)}$	Expected degradation in part-KC j bias
$b_{j\ell(uc)}$	Uncertainty in the bias in part-KC j
$\mathbf{b}_\ell$	Array of part-KC biases
n_f	Number of product-KCs
n_k	Number of KCs in layer k
s_t	Aperture step
$\sigma'_{j\ell}$	Maximum value of standard deviation in part-KC j
$\boldsymbol{\sigma_f}$	Array of product-KC standard deviations
σ_i	Standard deviation of product-KC i
$\sigma_{j(vr)}$	Standard deviation after variation reduction
σ_{jk}	Standard deviation of KC j in the k^{th} layer of the KC flowdown
$\sigma_{j\ell}$	Standard deviation of part-KC j
$\sigma_{j\ell(d)}$	Expected degradation in part-KC j standard deviation
$\sigma_{j\ell(uc)}$	Uncertainty in the standard deviation
$\boldsymbol{\sigma}_\ell$	Array of part-KC standard deviations
τ_1	Square of matrix δ_1
τ_k	Square of matrix δ_k
$\mathbf{T}$	Product of $(\ell-1)$ τ_k matrices
t_d	Thickness of door
T_{ij}	Element of matrix **T** relating variation in part-KC j to product-KC i.
UL_i	Upper tolerance limit of product-KC i
x_{jk}	Nominal value of KC j in layer k
x_r	X Roof location
y_a	Body location
y_d	Y location of door
y_r	Y Roof location